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**UNVEILING THE HIDDEN  
WORLD OF WOMEN IN KABULI  
SOCIETY THROUGH THE LENS  
OF TARAN KHAN'S**

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*Shadow City: A Woman walks Kabul*

**ABSTRACT**

This research aims to bring to the forefront the hidden world of women in Kabuli society and to focus on the undervalued contribution of women towards the development of the culture in the country. The research is framed to focus on two main theories i.e., cultural studies and feminism, thereby filling the research gap of the unspoken contribution of Kabuli women in building the culture of their country. Evidences are taken from the nonfiction work of Taran Khan, *Shadow City: A Woman Walks Kabul* to highlight the key contributions of women in fostering the culture of the country. From the lives of Kabuli queens to women martyrs, and Kabuli brides; women from various sections of the society are considered for the research. The research concludes with the finding that, the focus has to be given through research while studying female roles in society by suggesting different areas in which further research can be carried out.

**Keywords:** Women, Culture, Undervalud.

Afghanistan, a country in central Asia, has a rich heritage. Kabul, the capital city of the country is significantly noted for its resplendent culture, and traditional background along with its notable social, political, and economic background. The inhabitants of the capital city include people belonging to various ethnicities such as

Pashtuns, Tajiks, Hazaras, and Uzbeks, among others. The city has also been frequently prone to various wars that have affected the state of the country. Between 1990 to 2001 the city of Kabul was severely targeted as a battleground. The fall of the Taliban in 2001 called for reconstruction of the city and development towards progress thereafter.

Kabul is a city that is a hidden treasure that keeps unveiling itself under many discoveries. Being in a crucial period in Kabul after the Taliban regime, the author Taran Khan, despite her cautions of being warned not to tread through the streets of Kabul, entered those streets to unveil the mysteries. Her journeys began in 2006 and continued till 2013 which was far behind her expectations and she discovered a change in the city.

The writer Taran Khan is noted for her non-fictional, travel writings which is the theme of her first book, *Shadow City: A Woman Walks Kabul*, published in the year 2019. The book has been regarded as the Stanford Dolman Travel Book of the Year. Atiq Rahimi, a French-Afghan writer and filmmaker, and Alice Albinia an English journalist and author, have critically praised the book for its intricacies and wonderful portrayal of the culture, people, and past of Kabul.

The role of women has been crucial while considering their importance in shaping the traditions of countries in the world. They have made noteworthy contributions which can be seen through their influence in art, literature, politics, culture, and religion. Women have been the advocates of cultural heritage and in traditional societies, they pass down knowledge and skills through generations. In India, for example, women have passed down the tradition of the art of cooking to their future generations which forms the backbone of the culinary richness of the nation.

Women have a significant role in preserving the culinary tradition of Afghanistan. In ancient times when women were confined to the walls of the kitchen, they found a creative way of using the kitchen corners to construct recipes and delicacies like Kabuli Pulao that were introduced in the culinary history of Afghanistan. Women in Afghanistan also preserve the farming traditions of the country. Therefore, they gain a significant place in the development of culinary tradition.

Afghanistan culture and its development through the centuries have been shaped by the women in the country. Afghanistan women have marked their place in history by being the bearers of culture, values, beliefs, and tradition to pass them down through the ages. The development of arts and handicrafts in the country is owed to the efforts of women who have been masters of pottery, weaving, embroidery, and many intricate handiwork. Dance as an art form mainly developed through the women of Afghanistan

Many of the Contemporary female singers like Aryana Syeed, and Ghazal Sadat, have dedicated their talents to the development of Afghani music and marking a place for the country in the music world. Many women writers like Homeira Qaderi, and Layla Sarahat Rushani stand as an example of the victory gained by Afghan women in the literary world. In addition to these, women in Afghanistan have also emerged as successful political leaders, Elay Ershad., and Fawzia Koofi are a few to name. Thus, from a state of confinement, many Afghani women have stepped outside their homes and are achieving success in the world.

Women's rights have been a prevalent issue throughout the Afghani region. The Taliban regime became a nightmare especially while concerning women's rights. In 1990 when the Taliban were ruling Afghanistan,

they imposed strict laws upon women which prohibited them from getting an education, which is the basic right of every human being. They also banned women from working outside the house thereby leading to the decline of women force in the workplace. They also restricted the freedom of women by compelling them to wear burqas which was interference into the freedom of choice of dress of women.

There are several instances in the work *Shadow City* where Taran Khan talks about women and their roles and also unmask the harsh reality that is imposed upon the women of the society. This research paper aims to identify the real meaning behind every instance related to women as mentioned in the work and unveil the important yet hidden roles that women play in the Afghani society.

In the chapter Returns, Tarn Khan reveals the first historical incident in Afghanistan that sowed the seed of modernization. Queen Soraya and King Amanullah were the first royal pair who introduced the modernization measures. Women were restricted the freedom to dress according to their choice. But the queen broke these confines and rode on horseback wearing Western-style clothes. They also took a step to invite women to remove their veils. This had become a controversy.

In the chapter, Walking with the Djinn, Khan talks about the deteriorating mental health of women in Kabul. Women apart from being victims of abuse have also become martyrs. Farkhunda, a young woman who was attacked by a mob and who was considered a martyr was later revered respectfully by constructing a shrine in her memory. This gives us the idea that even in the time of suffering; women tend to inspire others to become an inspiration to many other women. This particular incident conveys the courage that women have, despite being held subservient to men.

The men in Kabul had been imposing several restrictions on women. They permitted cinemas for women only in a men-free zone. Therefore, the women in Kabul could enjoy cinema in the company of another woman. However, men, try to invade this personal space by disguising themselves as women covered in chadoris. Khan mentions one such incident, where a man disguised himself as a woman but was eventually caught. This incident questions the real freedom of entertainment for women, where men can easily hide themselves and sneak into an all-women crowd whereas when it comes to women, they had to be restricted in their freedom of choices.

In the chapter *Veiled City*, Khan discusses the role of women in Weddings. This is an important part of this research as it fully connects with the statement of bringing to light the hidden roles that women play in changing tradition and cultural practices based on their love for aesthetics and affinity towards change. From just agreeing to all ideas to deciding things on their own, women have gone a long way towards change.

Weddings and cultural practices have undergone many changes over the years. From having conservative weddings to having elaborate hall weddings, the changes in venues have marked many other significant changes. As discussed earlier, love was a secret relationship in Kabul while weddings were a grand announcement. They observed various traditions such as the etiquette of gift giving, having invites in the shape of sugar almonds, the mehendi ceremony, and much more.

However, women had no choice but to adhere to the rules of the weddings as to the choice of dresses which can be worn in front of everyone for the wedding and which can be worn only amidst close relatives. This however included grand frocks and sparkling dresses to sleeveless and low-backed gowns. These dresses though appear modern had their

fixed limits and boundaries within which they were to be worn.

The only choice that women had the freedom to make and the choice they made with utmost happiness was the makeup for the wedding. The cost of the makeup was to be borne by the groom's family. Though the women stylists were looked down upon by the men in the society, who even had to hide their work in the saloon from their husbands, took this risk every day and completed their duties of making the bride feel the most beautiful on their special day. The women also felt that they had their full freedom exercised and had been waiting all their lives for this particular phase of getting themselves ready for their weddings. We can also find that women enjoy themselves at weddings in private gatherings that have separate divisions for both sexes. Women danced their hearts out and had the best time of their lives. Khan mentions one such woman who danced around without even tending to her crying child, but this the woman calls a petty nuisance and remarks that she cannot waste her life on such silly things. This shows the eagerness of women waiting to express themselves and their ability to make the fullest of the chances they have.

Generally, women do not exercise their rights much owing to all the domestic hurdles they are subjected to. But Khan portrays simple, everyday actions in which women exhibit their simple acts of freedom and bring about remarkable changes in society. For example, the chapter on women deciding their appearance for weddings marks how long way women have come from being mere subjects during marriage to making some important decisions in it.

Khan's work unveils the hidden and overlooked creations of women which were brought into the frontier by their efforts but were often neglected. Khan brings to light these incidents thereby using them as evidence



in an act of disagreement toward the stereotypical ideologies imposed on women. This narrative helps in offering a comprehensive understanding of the changes in the culture of the country.

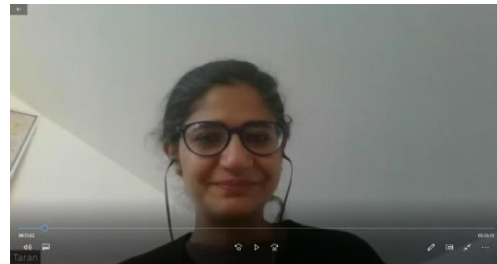
The work also brings to light the hardships that women undergo while moulding a country towards development and bringing it to the frontier. This situation reveals complicated struggles and threats through social norms and conflicts. The dominance of the patriarchal society, the restrictions imposed upon women, the impacts of war, and political tumults cause problems for women who strive for development. This work serves as a testimony of women who decided to overcome the barriers and mark their place in the history of Kabul in contributing towards its development.

In this nonfiction work, Khan emphasizes women who play an undeniable role in the development of the culture of a country. This role manifested by women has developed into a progressive and versatile force. Khan ventures into the convoluted intricacies of the urban life in Kabul where women maneuver a complicated interactivity between culture and resilience. Through her careful observations and commendable narrative technique mastering the nuances of storytelling, Khan brings to light the women of the Shadow City who despite being socially restricted, actively contribute towards knitting together the nation's cultural fabric.

This research further provides grounds for other research that can be carried out on various topics related to the current title. Research can be undertaken about identifying the role of women in shaping the political rule of Kabul, the role of women in contributing towards the economic development in Kabul, and the role of women in protecting the environment of Kabul amidst wars and overall

empowerment of Kabuli women.

In an interview conducted with the author of this work, Taran Khan, the author expressed her opinion that she was able to find many similarities and differences in the Kabuli culture. In the chapter about weddings and beauty parlors, the author was able to relate as the same is the happenings of the Indian culture during weddings. While considering the food during Kabuli weddings, Taran Khan found it fascinating with the variety of cuisine they had and the different traditions that they followed during the wedding.



*Screenshot from the Zoom meeting of an interview conducted by the researcher with the author*

To conclude, this research on highlighting the role of women in shaping the traditions and culture of Kabul has shed light on the complicated lives of Kabuli women and has unraveled the hidden meanings of the traditions in society and various cultures that have not been revealed to the world. Through careful examination and close reading into Tran Khan's *Shadow City* efforts have been made to bring to the frontier, the experiences of women who drive through fulfilling societal expectations while carrying the responsibility of protecting the legacy and overcoming the challenges of the era.

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## A PSYCHOSOCIAL INVESTIGATION OF AMIR’S CHARACTER EVOLUTION IN THE KITE RUNNER

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### ABSTRACT

Khaled Hosseini’s debut novel, *The Kite Runner* (2003), is a poignant and powerful exploration of human relationships, set against the backdrop of a tumultuous Afghan history. This article ventures on a panoramic character evaluation of the protagonist Amir in Khaled Hosseini’s *The Kite Runner*, applying Eric Erikson’s Psychosocial theory to illuminate the formation of a round character. This study inspects of the Amir’s developmental stages leveraging the Bildungsroman technique to cast light upon his metamorphosis. Through a nuanced application of Erikson’s theory, this research traverses Amir’s experiences, shaped by Afghanistan’s socio-political context and its influence on his character development. This investigation delves into the themes of guilt, redemption, and identity, underlining Amir’s relationships and experiences shaped by his own psyche. Furthermore, this study investigates the influence of historical events, cultural norms, and social expectations on Amir’s development, comprehending the reciprocation between individual growth and societal influences. By examining Amir’s character through the view point of psychosocial theory, this article offers insights into the convolution of human development, identity formation, and the indelible effects of childhood experiences.

**KEYWORDS:** Psychosocial development, Identity formation, Guilt and redemption, Afghan culture and politics, Coming-of-age, Bildungsroman.

## INTRODUCTION

Afghanistan's literary convention is one of the most affluent in central Asia, particularly Herat, the third-largest city of Afghanistan, an oasis city which is glorified as a centre of Persian scholarly and literary pursuit. The ancient works of Afghanistan Literature were orally dispatched, whereas the contemporary Literature is profound in the inheritance of the oral constitution and the predominantly written texts. In the twentieth century, Kabul, the capital of the country became the centre for literary figures and publishing.

Khaled Hosseini, born on March 4, 1965, in Kabul, Afghanistan, is an Afghan-American novelist, philanthropist, activist, and diplomat. Hosseini's novels are known for their authentic portrayals of Afghanistan, often featuring women as main characters who face male domination in the Islamic state. Hosseini's familiarity with Tajik and Pashtun cultures, as well as his childhood experiences of kite flying, inspired his debut novel, *The Kite Runner*. Published by Riverhead Books in 2003, it became a New York Times bestseller and earned him international acclaim.

Hosseini was recognized as a defender of Afghan refugee affairs. The social disruption activated his nomination as a goodwill ambassador in the year 2006 for the UN High Commission for Refugees. Hosseini established The Khaled Hosseini Foundation after visiting Afghanistan with UNHCR, which dispenses assistance to the needy people in Afghanistan. His debut novel, *The Kite Runner* (2003), tells a powerful story of friendship and redemption against the backdrop of a changing Afghanistan. It became an international bestseller and was adapted into a successful film.

His second novel, *A Thousand Splendid Suns* (2007), explores the lives of two Afghan women whose paths cross amid the

turmoil of war and societal oppression. The novel highlights themes of resilience and the struggle for personal freedom. Hosseini's third novel, *And the Mountains Echoed* (2013), is a multi-generational family saga that spans several decades and continents, illustrating how choices resonate through time and affect numerous lives. In addition to his novels, Hosseini has written short stories and essays, often reflecting on the plight of refugees and the impact of conflict. His literary works are celebrated for their emotional depth, compelling narratives, and rich depiction of Afghan life and culture.

## ERIK ERIKSON'S PSYCHOSOCIAL THEORY

Erik Erikson's psychosocial theory is a foundational model in developmental psychology, suggesting that personality evolves through eight distinct stages, each defined by a particular conflict that must be resolved. Unlike Freud, who focused on the role of sexuality in development, Erikson emphasized the impact of social and cultural influences, proposing that each stage involves a psychosocial crisis that shapes individual growth and identity.

The initial stage, Trust vs. Mistrust, spans from birth to one year and revolves around the infant's basic needs being met by caregivers. Consistent and reliable care fosters trust, while inconsistent care can lead to mistrust. The second stage, Autonomy vs. Shame and Doubt (ages 1-3), involves the child developing a sense of personal control over physical skills and independence. Success in this stage leads to autonomy, while failure results in feelings of shame and doubt.

The third stage, Initiative vs. Guilt (ages 3-6), is when children start to assert power and control over their environment through play and social interactions.

Successfully navigating this stage

results in a sense of initiative, while failure can cause feelings of guilt. The fourth stage, Industry vs. Inferiority (ages 6-12), takes place during the school years, where children face new social and academic demands. Achieving success leads to a sense of competence, whereas failure can result in feelings of inferiority.

During adolescence (ages 12-18), the fifth stage, Identity vs. Role Confusion, is critical for developing a personal identity and sense of self. Success in this stage fosters fidelity, the ability to adhere to societal standards and expectations, while failure can lead to role confusion and a weakened sense of self. The remaining stages—Intimacy vs. Isolation (young adulthood), Generativity vs. Stagnation (middle adulthood), and Integrity vs. Despair (late adulthood)—build upon earlier stages, focusing on relationships, productivity, and life reflection. Erikson's theory underscores that the personality development is a continuous process shaped by social and cultural factors throughout life.

## **PSYCHOSOCIAL EVOLUTION OF AMIR IN THE KITE RUNNER**

During the primitive stage of Trust versus Mistrust, Amir's early life is characterized by his father's unemotional nature, which fosters a deep-seated mistrust in Amir. Baba's failure to provide affection or emotional support leaves Amir feeling insecure and questioning his self-worth. According to Erikson, the first year of life is a period of rapid growth and development, and Amir's experiences during this formative time had shaped his future relationships.

Amir reflects, "I remember the way he used to look at me, like I was some kind of precious, exotic treasure" (Hosseini 15). This brings out Baba's detachment and its impact on Amir's trust issues. Amir's longing for his father's attention and approval is evident, but

Baba's emotional distance instills a sense of insecurity in him. This sense of mistrust influences Amir's relationships throughout the novel, particularly his relationship with Hassan.

The second stage is the stage of Autonomy versus Shame and Doubt, in which Amir's pursuit of independence is obstructed by his father's expectations, resulting in feelings of shame and self-doubt. Baba's persistent criticism and lack of approval make Amir feel inadequate, which hampers his sense of autonomy.

Erikson argues that, the child must learn to assert his will and test the limits of his environment, but Amir's efforts to assert himself are met with disapproval.

Amir reflects, "I was a coward, and I knew it" (Hosseini 67). This statement accentuates Amir's shame and self-doubt, rooted in his struggles to stand up to his father or pursue his own desires. His thwarted attempts at autonomy lead to a sense of powerlessness. Consequently, Amir's sense of shame and doubt impacts his relationships and decision-making throughout the story. His inability to assert himself erodes his confidence and plays a role in his eventual betrayal of Hassan.

In the next stage of Initiative versus Guilt, Amir's efforts to prove himself and win his father's approval led to significant guilt, as his actions driven by initiative result in severe consequences. Despite Erikson's assertion that children must learn to take initiative and assert his own desires, Amir's misguided actions cause harm rather than achieving his goals. Amir says "I had one last chance to make things right" (Hosseini 91). This quote shows Amir's awareness of his errors and his attempt to rectify them.

However, his efforts come too late, leaving him burdened with deep guilt. As a result, Amir is plagued by guilt throughout the

novel. His struggle to forgive himself leads to self-loathing and shapes his quest for redemption. The fourth stage deals with Industry versus Inferiority Amir's difficulties in school and his feelings of inferiority compared to Hassan contribute to his sense of inadequacy. His desire to prove himself and gain his father's approval drives him to compete with Hassan, but he continues to feel inferior.

"I was a mediocre student, and I knew it" (Hosseini 123). This line illustrates Amir's feelings of inadequacy and inferiority. His academic struggles and perceived shortcomings compared to Hassan deepen his self-doubt. Consequently, Amir develops a pervasive sense of inferiority that impacts his relationships and self-esteem throughout the novel. His difficulty in recognizing his own abilities and limitations leads to a lack of confidence.

Stage five deals with Identity versus Role Confusion, which describes Amir's time in America, along with his interactions with Soraya and Rahim Khan, plays a key role in his search for identity. He wrestles with balancing his Afghan heritage with his new American environment, which leads to confusion about his roles. Erikson asserts that, the adolescent must explore different identities and roles, and Amir's experiences in America act as a trigger for this exploration.

"I was an Afghan, but I was also an American" (Hosseini 215). This quote emphasizes Amir's difficulty in integrating his dual identities. His relationships with Soraya and Rahim Khan assist him in navigating this identity confusion.

As a result, Amir starts to form a clearer sense of identity, understanding his own strengths and weaknesses. This exploration of identity becomes a crucial part of his path toward redemption.

In the sixth stage of Intimacy versus

Isolation, Amir's quest for intimacy with Soraya and his efforts to find redemption through Hassan's son, Sohrab, emphasize the significance of deep, meaningful relationships. His longing for connection drives him to build close bonds, though his past errors risk pushing him into isolation. Amir's connections with Soraya and Sohrab were essential to his personal growth.

Amir reflects, "I had found a new family, a new home" (Hosseini 371) through which, Amir's search for intimacy and a sense of belonging becomes evident. His relationship with Soraya and Sohrab offer him both a sense of purpose and a place where he feels he belongs. As a result, Amir develops a deeper sense of intimacy and recognizes the value of significant relationships in his life.

His path toward forming these connections sets the stage for his journey towards forgiveness and redemption.

During the seventh stage of Generativity vs. Stagnation, Amir's attempts to atone for his past and find a sense of purpose illustrate the concept of generativity. His desire to make a positive difference motivates him to assist Sohrab and seek redemption. Erikson suggests that, the middle-aged adult must contribute to the society, and Amir's actions to make amends reflect this generative drive.

"I had found a new purpose" (Hosseini 421). This statement magnifies Amir's commitment to making a positive impact and giving back. His efforts to support Sohrab and pursue redemption are clear examples of generativity.

Hence, Amir develops a sense of generativity, understanding the significance of contributing to the greater good. This pursuit of generativity forms the basis for his eventual journey toward achieving integrity.

At the last stage of Integrity vs. Despair, Amir's contemplation of his life and pursuit of redemption reflects his struggle to

achieve integrity. He deals with his past errors and seeks forgiveness, which leads him toward a sense of integrity. Erikson observes that, the older adult must reflect on their life and find closure and Amir's path to redemption exemplifies this quest for integrity.

In the novel, Amir notes, "I had found some measure of peace" (Hosseini 441). This enhances Amir's pursuit of forgiveness and his effort to achieve integrity. His journey toward redemption and reconciliation demonstrates his progress toward integrity. Ultimately, Amir develops a sense of integrity by acknowledging the need for reflection and closure. This journey towards integrity offers him a sense of resolution and fulfilment.

Erikson's eight stages of life would aid an individual to discover their own identity and individuality. The psychological aspect of the theory proves that the society has a greater role to play in the process of psychological development of a character. The society shaped Amir's psychological structure by forming and deforming several aspects in Amir through various social and situational incidents. Hence Amir was both affected and healed by the society. Erik Erikson's theory was applicable in almost all the stages of Amir's life and therefore Amir's psychosocial development becomes clearly visible under the light of psychosocial theory.

## CONCLUSION

In conclusion, this analysis reinforces the relevance of Erik Erikson's psychosocial theory in exploring the intricacies of human development as portrayed in Khaled Hosseini's *The Kite Runner*. Through Amir's journey, Hosseini vividly illustrates Erikson's eight developmental stages, highlighting the challenges and achievements inherent in personal growth. Amir's progression through the stages—from Trust vs. Mistrust to Integrity vs. Despair—serves as a model for understanding the impact of social interactions

and relationships on identity formation. The novel emphasizes the importance of resolving each developmental crisis, demonstrating how unresolved conflicts can lead to feelings of guilt, shame, and inadequacy. *The Kite Runner* ultimately conveys that redemption and forgiveness are attainable despite past transgressions. Amir's quest for integrity reflects the potential for human growth, self-awareness, and transformation. Moreover, the study highlights the continued relevance of Erikson's theory in psychological research, suggesting avenues for future exploration of psychosocial concepts within literary analysis. The insights gained from this analysis can enhance the application of psychological frameworks to literary studies, enriching our understanding of the human experience.

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## FIXED POINT THEOREMS FOR RATIONAL TYPE-CONTRACTION MAPPINGS IN $\mathcal{B}$ -METRIC SPACES

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### ABSTRACT

In this article basic definitions of metric spaces and  $\mathcal{B}$ -metric spaces are presented. Also we prove the context of  $\mathcal{B}$ -metric spaces, fixed points of rational type contractions also exist. Additionally, we provide some instances to show the reliability of the findings in the research. In the end, the results are used to solve an integral problem.

### INTRODUCTION

Fixed point theorems are developed for single-valued or set-valued mappings of abstract  $\mathcal{M}$  metric spaces. Fixed point results in  $\mathcal{B}$ - $\mathcal{M}$  metric spaces provide a generalization of fixed point theorems in  $\mathcal{M}$  metric spaces. In particular, rational type contraction mappings are class of mappings that generalize contraction mappings. In  $\mathcal{B}$ - $\mathcal{M}$  metric spaces, rational type contraction mappings ensure the

existence and uniqueness of fixed points under certain conditions. An application of fixed point results of rational type contraction mappings in  $\mathcal{b}$ - $\acute{m}$  etric spaces can be found in various areas of mathematics. Overall, fixed point results of rational type contraction mappings in  $\mathcal{b}$ - $\acute{m}$  etric spaces provide a powerful mathematical tool with diverse applications in various fields such as economics, computer science and analysis.

One of the most well-known and well-established theories in mathematics, fixed point theory has many applications. According to this theory, contraction is a key instrument for demonstrating the uniqueness and existence of a fixed point. In 1922, Banach introduced one of the basic and most widely applied fixed point theorems in Banach contraction mapping principle. Many authors proved the Banach contraction principle in different generalized  $\acute{m}$  etric spaces. Bakhtin first proposed the idea of  $\mathcal{b}$ - $\acute{m}$  etric space in 1989. Czerwik expanded on the findings of  $\mathcal{b}$ - $\acute{m}$  etric spaces in 1993.

## PRELIMINARIES

### Definition 1.1.1

Consider the non-empty set  $\mathcal{Y}$ . If  $\vartheta: \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}$  maintains the following characteristics, it is referred to as a  $\acute{m}$  etric space or a distance function on  $\mathcal{Y}$ ,

- i.  $(\alpha, \beta) \geq 0$  for all  $\alpha, \beta \in \mathcal{Y}$
- ii.  $(\alpha, \beta) = 0$  iff  $\alpha = \beta$
- iii.  $\alpha, \beta = (\beta, \alpha)$  for all  $\alpha, \beta \in \mathcal{Y}$  iv.  $\vartheta(\alpha, \beta) \leq \vartheta(\alpha, i) + \vartheta(i, \beta)$  for all  $\alpha, \beta, i \in \mathcal{Y}$

Then the ordered pair  $(\mathcal{Y}, \vartheta)$  is called a ***metric space***.

### Definition 1.1.2

When  $(\alpha, \beta) = |\alpha - \beta|$  in a real number ( $\mathbb{R}$ ),  $\vartheta$  is referred to be a  $\acute{m}$  etric on real number. We refer to this statistic as a  $\acute{m}$  etric in real numbers. We refer to this  $\acute{m}$  etric as

the ***usual metric***.

### Definition 1.1.3

Let a  $\acute{m}$  etric space be  $(\mathcal{Y}, \vartheta)$ . A set  $(\alpha)$  made up of all points  $\beta \in \mathcal{Y}$  such that  $(\beta, \alpha) < r$  is considered the ***neighbourhood*** of a point,  $\alpha \in \mathcal{Y}$ . The radius of  $(\alpha)$  is the actual number  $r$ .

$$(i.e) (\alpha) = \{\beta \in \mathcal{Y} / (\alpha, \beta) < r\}$$

$$(i.e) n \in (\alpha) \text{ which implies } (\alpha, \beta) < r.$$

### Example 1.1.4

An open interval  $(g, h)$  is a ***neighbourhood*** of each of its points on the real line.

### Definition 1.1.5

Assume  $\mathcal{K} \subset X$  and that  $(\mathcal{Y}, \vartheta)$  is a  $\acute{m}$  etric space. If there is a ***neighbourhood***  $\mathcal{Y}$

such that  $\mathcal{Y} \subset D$  for a point  $s$ , then  $s$  is an ***interior point*** of  $\mathcal{K}$ .

### Example 1.1.6

Consider the real number with the ***usual metric***. Assume that  $\mathcal{W} = (2,3) \cup \{4\}$ . Any point inside  $\mathcal{W}$  is located between 2 and 3. However, 4 is not a point inside  $\mathcal{W}$ . Due to the fact that we are unable to locate any ***neighbourhood*** of 4 inside  $\mathcal{W}$ .

### Definition 1.1.7

Given a  $\acute{m}$  etric space  $(\mathcal{Y}, \vartheta)$ , let  $J$  be a subset of  $\mathcal{Y}$ . If all of  $J$ 's points are interior points, then  $J$  is ***open***.

### Example 1.1.8

Assume the real number subset  $B = (3,4)$ . Consider the  $\acute{m}$  etric space  $(\mathcal{Y}, \vartheta)$  with usual  $\acute{m}$  etrics.



$\therefore B = (3,4)$  is an *open set*.

**Definition 1.1.9**

Let  $J$  be a subset of  $\mathcal{Y}$  and  $(\mathcal{Y}, \vartheta)$  be a  $m'$  etric space. If there are points of  $J$  that are different from  $s$  in each of  $s$ 's *neighbourhood*, then  $s$  is referred to as a **limit point** of the set  $J$ .

**Example 1.1.10**

Consider the  $m'$  etric space  $(\mathcal{Y}, \vartheta)$ . Let  $\mathcal{W}$  be the subset of  $\mathcal{Y}$ , with  $\mathcal{W} = (0,1)$ . In this case,  $M$ 's limit point is  $0,1$ . Additionally, every point that  $S \subset \cup_{i=1}^n M_{\delta_i}$  lies between

the finite subcollection  $\{M_{\delta_1}, M_{\delta_2}, \dots \dots \dots M_{\delta_n}\}$ .

**Definition 1.1.11**

Let  $\mathcal{H} \subset \mathcal{Y}$  be a  $m'$  etric space and let  $(\mathcal{Y}, \vartheta)$  be the space. The collection of all interior points is denoted by  $(\mathcal{H})$ .

**Definition 1.1.12**

Let  $(\mathcal{Y}, \vartheta)$  be a  $m'$  etric space and let  $\mathcal{K} \subset \mathcal{Y}$ . A point  $x \in \mathcal{Y}$  is called an

**isolated point** of  $\mathcal{K}$  if it is a limit point of .

**Definition 1.1.13**

Consider the  $m'$  etric space  $(\mathcal{Y}, \vartheta)$ . Assume  $V \subset \mathcal{Y}$ . If every limit point in  $V$  is also a point in  $V$ , then  $V$  is said to be the **closed set**.

**Example 1.1.14**

Let  $= [2,3] \subset \mathbb{R}$

All real numbers between 2 and 3 and the limit point  $\mathcal{K} = 2,3$  are represented by the point  $\mathcal{K} = [2,3]$ . Every closed interval is closed.

**Definition 1.1.15**

Consider  $(\mathcal{Y}, \vartheta)$  be a  $m'$  etric space. Then  $Z \subseteq E$ . Then  $\mathcal{M}$  is said to be **perfect set** if i.  $\mathcal{M}$  is closed.

ii. Every point in  $\mathcal{M}$  is also its limit.

**Example 1.1.16**

A perfect set consists of only one closed set.

**Definition 1.1.17**

If the  $m'$  etric space  $(\mathcal{Y}, \vartheta)$  is both complete and entirely bounded, then it is considered as **compact**.

**Example 1.1.18**

Finite sets are all compact.

**Definition 1.1.19**

If every Cauchy sequence of points in a space  $R$  converges to a single point in the space, then the space  $R$  is said to be **complete**.

**Example 1.1.20**

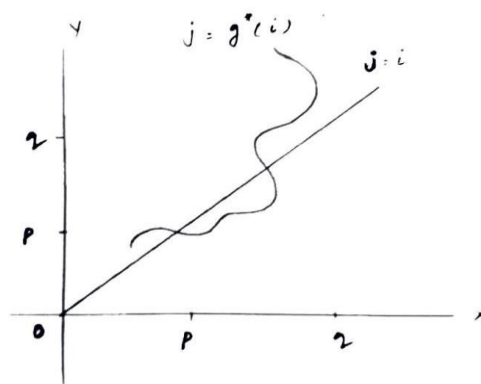
Any  $m'$  etric space that is discrete is complete.

**Definition 1.1.21**

The sequence  $\{p_n\}$  in a  $m'$  etric space  $(\mathcal{Y}, \vartheta)$  is called **Cauchy sequence**. For every  $\epsilon > 0$  and there exist integer  $N$  such that  $|p_n - p_m| < \epsilon$  for all  $n, m \geq N$ .

**1.2 FIXED POINT TEOREM**

Assume the graph of a continuous real function  $b^*: [s, v] \rightarrow [s, v]$ . Hence that, continuous simply means that the graph is in one piece and the fact that  $^*: [s, v] \rightarrow [s, v]$ , and means that it is defined or all  $t$  with  $s \leq t \leq v$  and that  $b^*(t)$  satisfies  $s \leq b^*(t) \leq v$ .



Assume the line (since  $b^*(s)$  being in  $[s, v]$ , is atleast  $p$ ) when  $t = v$ . The Graph  $b^*$  is on or below the line (since  $b^*(v)$  being in  $[s, v]$ , is atmost  $q$ ). Since the graph of  $b^*$  is in one piece, it means that at some point in  $[k, l]$  the graph of  $b^*$  crosses the line. That is for some  $t \in [s, v]$ , we have  $t = b^*(t)$ .

**Definition 1.2.1**

Let  $\mathcal{M}: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping and  $(\mathcal{Y}, \vartheta)$  be a  $\acute{m}$  etric space. If  $g$  is mapped onto itself, then the point  $g \in \mathcal{Y}$  is referred to as a **Fixed point** of  $\mathcal{M}$ . That is  $\mathcal{M}g = g$ .

**Example 1.2.2**

- There is no fixed point in a translational.
- A random point on the plane has one fixed point. The only fixed point is, in fact, the rotation's center.
- Two fixed points exist in the mapping of  $\mathbb{R}$  into  $t \rightarrow t^2$ . The fixed points are, in fact, 0 and 1.
- There are an infinite number of fixed points in the  $\mathbb{R}^2$  projection  $(p, q) \rightarrow p$  onto the x-axis. The x-axis's points are completely fixed, in actuality.

As seen by the example in 1.2.2, a mapping might not have a fixed point at all. It could have one fixed point, several fixed points, or even an infinite number of fixed points.

**Definition 1.2.3**

Let  $(\mathcal{Y}, \vartheta)$  be a  $\acute{m}$  etric space.  $K: \mathcal{Y} \rightarrow \mathcal{Y}$  is said to be **non expansive** if

$$(Km, Kn) \leq (m, n) \text{ for every } m, n \in \mathcal{Y}.$$

**Definition 1.2.4**

Let  $(\mathcal{Y}, \vartheta)$  be a  $\acute{m}$  etric space and  $\mathcal{K}: \mathcal{Y} \rightarrow \mathcal{Y}$  is called **contraction** of  $\mathcal{Y}$  if there exists a real number  $\delta$  with  $0 \leq \delta < 1$  such that  $(K\alpha, K\beta) \leq \delta(\alpha, \beta)$  for all  $\alpha, \beta \in \mathcal{Y}$  and  $\alpha \neq \beta$ . The number  $\delta$  is called Lipschitz constant of  $K$ .

**Note 1.2.5**

- ❖  $\mathcal{K}$  is a contraction  $\implies \mathcal{K}$  is contractive  $\implies \mathcal{K}$  is non expansive. In spite of that the converse is not true.
- ❖ For example, the identity map  $\mathcal{K}: I \rightarrow I$  where  $I$  is any  $\acute{m}$  etric space, is non expansive but not contractive.

**Theorem 1.2.6 (Banach fixed point theorem)**

Consider  $(\mathcal{Y}, \vartheta)$  be a complete  $\acute{m}$  etric space and the mapping  $L: \mathcal{Y} \rightarrow \mathcal{Y}$  be a contraction on  $\mathcal{Y}$ . Then,  $L$  has a unique fixed point in  $\mathcal{Y}$ .

**Proof:**

Let  $\delta \in [0,1]$  be the Lipschitz constant thus  $\vartheta(Wx, Wy) \leq \delta\vartheta(x, y)$  for all  $x, y \in X$ . We prove the theorem in various steps.

**Step(i):** We construct a sequence  $\{h_m\} \subset \mathcal{Y}$  as follows.

Take any point  $h_0 \in \mathcal{Y}$  and inductively construct the sequence  $\{h_m\}$  of points in  $\mathcal{Y}$  as follows.

$$\begin{aligned}
 h_1 &= h_0 \\
 h_2 &= Wh_1 = W^2h_0 \\
 h_3 &= Wh_2 = W^3h_0 \dots \dots \dots \dots \dots \dots \\
 &\dots \dots \dots \dots \dots
 \end{aligned}$$

$$h_m = Wh_{m-1} = W^mh_0$$

Clearly  $\{h_m\}$  is the sequence of images of  $h_0$  under repeated application of  $W$ .

**Step (ii):**  $\{h_m\}$  is a Cauchy sequence in  $\mathcal{Y}$ .

Let  $n < m$ . Then

$$\begin{aligned}
 (h_n, h_m) &= (W^n h_0, W^m h_0) \\
 (h_n, h_m) &= (W^{n-1} h_0, W^{m-1} h_0) \\
 &\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\
 &\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\
 (h_n, h_m) &\leq (h_0, W^{m-n} h_0)
 \end{aligned}$$

$$\begin{aligned} &\leq [(h_0, Wh_0) + (h_0, Wh_0) + \dots + \\ \delta^{m-n-1}(h_0, Wh_0)] \\ &\leq [1 + \delta + \delta^2 + \dots + \delta^{m-n-1}](h_0, Wh_0) \\ \vartheta(h_n, h_m) &\leq \frac{\delta^n}{1-\delta} \vartheta(h_0, Wh_0) \quad , (0 \leq \delta \leq 1) \\ &= 0 \end{aligned}$$

As  $m \rightarrow \infty$ , hence  $\{h_m\}$  is a Cauchy sequence.

**Step (iii):**

Since  $\mathcal{Y}$  is complete and  $\{h_m\}$  is a Cauchy sequence in  $\mathcal{H}$ .  $h \in \mathcal{H}$  such that  $h_m \rightarrow h$ .

**Step (iv):**  $h$  is a fixed point of  $W$ .

We have,

$$\begin{aligned} (h, Th) &\leq (h, h_m) + (h_m, Wh) \text{ (by triangle} \\ \text{inequality)} \\ &\leq (h, h_m) + (Wh_{m-1}, Wh) \\ &= 0 \text{ as } n \rightarrow \infty \end{aligned}$$

$$(h, Th) = 0$$

$$\therefore h = Wh$$

Hence  $h$  is a fixed point of  $W$ .

Thus the existence of a fixed point is established. Now we shall verify that the uniqueness of such a fixed point.

**Step (v):**  $h$  is a unique fixed point of  $W$ .

Let  $h$  and  $i$  be the two fixed point of  $W$  in  $H$ . Then  $h = Wh$  and  $i = Wi$ .

$$\vartheta(h, i) = \vartheta(Wh, Wi) \leq \delta \vartheta(h, i)$$

$$(h, i) = 0$$

$$\therefore h = i$$

This complete the proof.

**Note 1.2.7**

The above theorem is also called as Banach contraction theorem.

**Example 1.2.8**

Take  $\mathcal{H} = [0, \frac{1}{2}]$  equipped with the  $m'$  etric of absolute value. This is known as incomplete

$m'$  etric space. Hence that the mapping  $\mathcal{W}: \mathcal{H} \rightarrow \mathcal{H}$  given by  $Wh = h^2$  is a contraction but  $\mathcal{W}$  has no fixed point.

**Remark 1.2.9**

The condition of  $\mathcal{K}$  being a contraction cannot be replaced by the weaker one, namely contractive.

**Example 1.2.10**

Consider the complex  $m'$  etric space  $\mathcal{H} = [0, \infty]$  equipped with the  $m'$  etric of absolute value and consider the mapping  $\mathcal{K}: \mathcal{H} \rightarrow \mathcal{H}$  given by  $\mathcal{K}h = \frac{1}{1+h^2}$  then,

- (a) The mapping  $\mathcal{K}$  satisfies  $(\mathcal{K}h, \mathcal{K}i) \leq (h, i)$  and hence  $\mathcal{K}$  is a contractive map, while  $\mathcal{K}$  is not a contraction.
- (b) There is no fixed point for  $\mathcal{K}$ .

**Theorem 1.2.11**

Consider  $(\mathcal{Y}, \vartheta)$  be a compact  $m'$  etric space and  $K: \mathcal{Y} \rightarrow \mathcal{Y}$  a contractive map. Then  $K$  has a unique fixed point in  $\mathcal{Y}$ .

**Proof:**

Define  $a^*: \mathcal{Y} \rightarrow \mathbb{R}$  by

$a^*(h) = \vartheta(h, Kh)$  we first established that  $a^*$  is continuous.

Let  $\epsilon > 0$  be given. Then

$$\begin{aligned} |a^*(h) - a^*(i)| &= |\vartheta(h, Kh) - \vartheta(i, Ki)| \\ &\leq |\vartheta(h, Kh) - \vartheta(i, Ki)| + \vartheta(h, Kh) - \vartheta(i, Ki) \\ &\leq \vartheta(h, i) + \vartheta(Kh, Ki) \\ &\leq 2(h, i) \\ &= \frac{\epsilon}{2}. \end{aligned}$$

**FIXED POINT THEOREMS ON  $\mathcal{B}$ - $M'$ ETRIC SPACES**

**Definition 2.1**

Consider the set  $\mathcal{Y}$  and let  $s \geq 1$  be a real number. A function  $\vartheta: \mathcal{Y} \times \mathcal{Y} \rightarrow R^+$  is said to be  $\mathcal{b}$ - $\acute{m}$  etric iff for all  $\alpha, \beta, \gamma \in \mathcal{Y}$  the following conditions are satisfied:

- 1)  $(\alpha, \beta) = 0$  if and only if  $\alpha = \beta$ ;
- 2)  $(\alpha, \beta) = \vartheta(\beta, \alpha)$ ;
- 3)  $(\alpha, \gamma) \leq s \cdot [\vartheta(\alpha, \beta) + \vartheta(\beta, \gamma)]$ .

Hence the pair  $(\mathcal{Y}, \vartheta)$  is called a  $\mathcal{b}$ - $\acute{m}$  etric space.

### Definition 2.2

Let  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ - $\acute{m}$  etric space. Then, a sequence  $\{y_n\}$  in  $\mathcal{Y}$  is called:

1. **convergent** if there exists  $u \in \mathcal{Y}$  such that  $\vartheta(\alpha_n, u) \rightarrow 0$ , as  $n \rightarrow +\infty$ . In this case we write  $\lim_{n \rightarrow \infty} \alpha_n = u$ .
2. **cauchy** iff  $\vartheta(\alpha_n, \alpha_m) \rightarrow 0$ , as  $n, m \rightarrow +\infty$ .

### Definition 2.3

The  $\mathcal{b}$ - $\acute{m}$  etric space  $(\mathcal{Y}, \vartheta)$  is said to be **complete** if every Cauchy sequence in  $\mathcal{Y}$  converges in  $\mathcal{Y}$ .

### Lemma 2.4

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping. Suppose that  $\{\alpha_n\}$  is a sequence in  $\mathcal{Y}$  induced by  $\alpha_{n+1} = T\alpha_n$  such that

$$(\alpha_n, \alpha_{n+1}) \leq \Delta(\alpha_{n-1}, \alpha_n) \quad \forall n \in \mathbb{N}$$

Where  $\Delta \in [0,1)$  is a constant. Then  $\{\alpha_n\}$  is a Cauchy sequence.

### Theorem 2.5

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ -complete  $\mathcal{b}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\alpha) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\beta)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} \quad (2.1)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2 \geq 0, (\alpha, T\beta) + (\beta, T\alpha) \neq 0$  with  $\Delta_1 + \Delta_2 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

### Theorem 2.6

Let  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ -complete  $\mathcal{b}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\alpha)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} + \Delta_3 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\alpha) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\beta)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} \quad (2.2)$$

Where  $\Delta_1, \Delta_2, \Delta_3$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

### Theorem 2.7

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ -complete  $\mathcal{b}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and

$T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)} + \Delta_3 \frac{\vartheta(\alpha, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, \beta)} + \Delta_4 [\vartheta(\alpha, T\alpha) + \vartheta(\beta, T\beta)] + \Delta_5 [\vartheta(\beta, T\alpha) + \vartheta(\alpha, T\beta)] \quad (2.3)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3, \Delta_4$  and  $\Delta_5$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 + 2\Delta_4 + 2s\Delta_5 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

### Theorem 2.8

Consider be a  $\mathcal{b}$ -complete  $\mathcal{b}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)} + \Delta_3 \frac{\vartheta(\beta, T\beta)[1 + \vartheta(\alpha, T\alpha)]}{1 + \vartheta(\alpha, \beta)} \quad (2.4)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

### Theorem 2.9

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{L}$ -complete  $\mathcal{L}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\begin{aligned} \vartheta(T\alpha, T\beta) \leq & \Delta_1\vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)} + \Delta_3 \frac{\vartheta(\alpha, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, \beta)} \\ & + \Delta_4 \frac{\vartheta(\beta, T\beta)[1 + \vartheta(\alpha, T\alpha)]}{1 + \vartheta(\alpha, \beta)} \end{aligned} \quad (2.5)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3$  and  $\Delta_4$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

### Theorem 2.10

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{L}$ -complete  $\mathcal{L}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\begin{aligned} \vartheta(T\alpha, T\beta) \leq & \Delta_1\vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\alpha, T\beta) + \vartheta(\beta, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} \\ & + \Delta_3\vartheta(T\alpha, T\beta) \end{aligned} \quad (2.6)$$

For all  $\mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3$  are nonnegative constants,  $(\alpha, T\beta) + (\beta, T\alpha) \neq 0$  with  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

## FIXED POINT THEOREMS FOR RATIONAL TYPE - CONTRACTION MAPPINGS IN $\acute{M}$ ETRIC SPACES

The *Banach* Contraction Principle, also known as the *Banach* Fixed-Point Theorem, was formulated by the Polish mathematician Stefan *Banach* in the early 20th century. *Czerwik*, has worked on extensions and generalizations of the *Banach* Contraction Principle, especially in the context of  $\mathcal{L}$ - $\acute{m}$  etric spaces, which are generalizations of  $\acute{m}$  etric spaces. In the context of  $\acute{m}$  etric fixed point theory, the *Banach* Contraction Principle serves as a foundational tool for proving the existence and uniqueness of fixed points for mappings defined on  $\acute{m}$  etric spaces. It provides a rigorous framework for studying the behavior of iterative processes and establishing convergence properties.

Fixed-point theorems are essential results in mathematics that establish conditions under which solutions to certain equations or mappings exist. In the context of contraction mappings, which are mappings that contract distances between points, there are several fixed-point theorems. One of the most well-known is the *Banach* Fixed-Point Theorem, which applies to complete  $\acute{m}$  etric spaces. When discussing rational-type contraction mappings, we are often referring to mappings that involve rational functions. However, fixed-point theorems for such mappings might vary depending on the context and the properties of the mappings involved.

A rational-type contraction mapping, sometimes referred to as a rational contraction mapping, is a concept in mathematics, particularly in the theory of dynamical systems and functional analysis. It builds upon the notion of a contraction mapping, which is fundamental in the study of fixed points and iterative methods.

The existence and uniqueness of fixed points in rational-type contraction mappings would depend on the specifics of the mapping, the space it operates on, and the contraction properties of the mapping.

### Theorem-3.1

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{L}$ -complete  $\mathcal{L}$ - $\acute{m}$  etric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$T: \mathcal{Y} \rightarrow \mathcal{Y}$$

$$\vartheta(T\alpha, T\beta) \leq \Delta_1\vartheta(\alpha, \beta)$$

$$+ \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\alpha) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\beta)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} + \Delta_3 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)}, \quad (3.1)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

*Proof:*

Let  $\alpha_0$  be arbitrary in  $\mathcal{Y}$ , we define a sequence  $\{\alpha_n\}$  in  $\mathcal{Y}$  such that  $\alpha_{n+1} = T\alpha_n$ , for all  $n \in \mathbb{N}$ , from the condition (3.1) with  $\alpha = \alpha_{n-1}$ ,  $\beta = \alpha_n$  and  $T\alpha_{n-1} = \alpha_n$ .

Therefore

$$(\alpha_n, \alpha_{n+1}) = (T\alpha_{n-1}, T\alpha_n)$$

$$\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, T\alpha_{n-1})\vartheta(\alpha_n, T\alpha_{n-1}) + \vartheta(\alpha_n, T\alpha_n)\vartheta(\alpha_{n-1}, T\alpha_n)}{\vartheta(\alpha_{n-1}, T\alpha_n) + \vartheta(\alpha_n, T\alpha_{n-1})} +$$

$$\Delta_3 \frac{\vartheta(\alpha_{n-1}, T\alpha_{n-1})\vartheta(\alpha_n, T\alpha_n)}{\vartheta(\alpha_{n-1}, \alpha_n)}$$

$$\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_n, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_{n-1}, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_n)} +$$

$$\Delta_3 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_n, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_n)}$$

$$\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_{n-1}, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_{n+1})} + \Delta_3 \vartheta(\alpha_n, \alpha_{n+1})$$

$$\leq \Delta_1(\alpha_{n-1}, \alpha_n) + \Delta_2 \vartheta(\alpha_n, \alpha_{n+1}) + \Delta_3 \vartheta(\alpha_n, \alpha_{n+1})$$

It follows that

$$(1 - \Delta_2 - \Delta_3)(\alpha_n, \alpha_{n+1}) \leq \Delta_1(\alpha_{n-1}, \alpha_n) \quad (3.2)$$

$$\vartheta(\alpha_n, \alpha_{n+1}) \leq \left( \frac{\Delta_1}{1 - \Delta_2 - \Delta_3} \right) \vartheta(\alpha_{n-1}, \alpha_n)$$

Put  $\Delta = \frac{\Delta_1}{1 - \Delta_2 - \Delta_3}$ . In view of  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ , then  $0 \leq \Delta < 1$ . Then by lemma 2.5,  $\{\alpha_n\}$  is a  $\mathcal{b}$ -Cauchy sequence in  $\mathcal{Y}$ . Since  $(\mathcal{Y}, \vartheta)$  is  $\mathcal{b}$ -complete, there exists some point  $u^* \in \mathcal{Y}$  such that  $\alpha_n \rightarrow u^*$  as  $n \rightarrow \infty$ .

By (3.1), it's easy to see that

$$(\alpha_{n+1}, u^*) = (T\alpha_n, Tu^*) \quad (3.3)$$

$$\leq \Delta_1 \vartheta(\alpha_n, u^*) + \Delta_2 \frac{\vartheta(\alpha_n, T\alpha_n)\vartheta(u^*, T\alpha_n) + \vartheta(u^*, Tu^*)\vartheta(\alpha_n, Tu^*)}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, T\alpha_n)} +$$

$$\Delta_3 \frac{\vartheta(\alpha_n, T\alpha_n)\vartheta(u^*, Tu^*)}{\vartheta(\alpha_n, u^*)}$$

$$\leq \Delta_1 \vartheta(\alpha_n, u^*) + \Delta_2 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(u^*, \alpha_{n+1}) + \vartheta(u^*, Tu^*)\vartheta(\alpha_n, Tu^*)}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, \alpha_{n+1})} +$$

$$\Delta_3 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(u^*, Tu^*)}{\vartheta(\alpha_n, u^*)} \quad (3.4)$$

Taking the limit as  $n \rightarrow \infty$  by both components of (3.4), then we have

$$\lim_{n \rightarrow \infty} \vartheta(\alpha_{n+1}, Tu^*) = 0.$$

That's,  $\alpha_n \rightarrow Tu^*$ .

Thus,  $Tu^* = u^*$ ,  $u^*$  is a fixed point of  $T$ . In

the end, we prove the uniqueness of the fixed point. Indeed, if there is another fixed point  $v^*$ , by (3.1) we have,

$$\vartheta(u^*, v^*) = \vartheta(Tu^*, Tv^*)$$

$$\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, Tu^*)\vartheta(v^*, Tu^*) + \vartheta(v^*, Tv^*)\vartheta(u^*, Tv^*)}{\vartheta(u^*, Tv^*) + \vartheta(v^*, Tu^*)} +$$

$$\Delta_3 \frac{\vartheta(u^*, Tu^*)\vartheta(v^*, Tv^*)}{\vartheta(u^*, v^*)}$$

$$\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, u^*)\vartheta(v^*, u^*) + \vartheta(v^*, v^*)\vartheta(u^*, v^*)}{\vartheta(u^*, v^*) + \vartheta(v^*, u^*)} + \Delta_3 \frac{\vartheta(u^*, u^*)\vartheta(v^*, v^*)}{\vartheta(u^*, v^*)}$$

$$\vartheta(u^*, v^*) \leq \Delta_1 \vartheta(u^*, v^*)$$

Since  $\Delta_1 + \Delta_2 + \Delta_3 < 1$  implies  $\Delta_1 < 1$ , we obtain that  $\vartheta(u^*, v^*) = 0$ , i.e.,  $u^* = v^*$ .

### Theorem-3.2

Consider  $(\mathcal{Y}, \vartheta)$  be a  $\mathcal{b}$ -complete  $\mathcal{b}$ - $m$  metric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\alpha, T\beta) + \vartheta(\beta, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} +$$

$$\Delta_3[(\alpha, T\alpha) + (\beta, T\beta)] \quad (3.6)$$

For all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ . Hence T has a unique fixed point in  $\mathcal{Y}$ .

*Proof:*

Choose  $\alpha_0 \in \mathcal{Y}$  and construct a Picard iterative sequence  $\{\alpha_n\}$  by  $\alpha_{n+1} = T\alpha_n$ . If there exists  $n_0 \in \mathbb{N}$  thus  $\alpha_{n_0} = \alpha_{n_0+1}$ , then  $\alpha_{n_0} = \alpha_{n_0+1} = T\alpha_{n_0}$ , i.e.,  $\alpha_{n_0}$  is a fixed point of T. Afterwards, without loss of generality, let  $\alpha_n \neq \alpha_{n+1}$  for all  $n \in \mathbb{N}$ . By (3.6), we get  $(\alpha_n, \alpha_{n+1}) = (T\alpha_{n-1}, T\alpha_n)$

$$\begin{aligned} &\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, T\alpha_{n-1})\vartheta(\alpha_{n-1}, T\alpha_n) + \vartheta(\alpha_n, T\alpha_n)\vartheta(\alpha_n, T\alpha_{n-1})}{\vartheta(\alpha_{n-1}, T\alpha_n) + \vartheta(\alpha_n, T\alpha_{n-1})} + \\ &\quad \Delta_3 [\vartheta(\alpha_{n-1}, T\alpha_{n-1}) + \vartheta(\alpha_n, T\alpha_n)] \\ &\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_{n-1}, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_n, \alpha_n)}{\vartheta(\alpha_{n-1}, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_n)} + \\ &\quad \Delta_3 [\vartheta(\alpha_{n-1}, \alpha_n) + \vartheta(\alpha_n, \alpha_{n+1})] \\ &\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_{n-1}, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_{n+1})} + \\ &\quad \Delta_3 [\vartheta(\alpha_{n-1}, \alpha_n)] + \Delta_3 [\vartheta(\alpha_n, \alpha_{n+1})] \\ &\leq \Delta_1 (\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_{n-1}, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_{n+1})} + \\ &\quad \Delta_3 [\vartheta(\alpha_{n-1}, \alpha_n)] + \Delta_3 [\vartheta(\alpha_n, \alpha_{n+1})] \\ &\leq \Delta_1 (\alpha_{n-1}, \alpha_n) + \Delta_2 \vartheta(\alpha_{n-1}, \alpha_n) + \\ &\quad \Delta_3 [\vartheta(\alpha_{n-1}, \alpha_n)] + \Delta_3 [\vartheta(\alpha_n, \alpha_{n+1})] \\ (1 - \Delta_3)(\alpha_n, \alpha_{n+1}) &\leq (\Delta_1 + \Delta_2 + \Delta_3)(\alpha_{n-1}, \alpha_n) \\ \vartheta(\alpha_n, \alpha_{n+1}) &\leq \left( \frac{\Delta_1 + \Delta_2 + \Delta_3}{1 - \Delta_3} \right) \vartheta(\alpha_{n-1}, \alpha_n) \end{aligned}$$

Put  $\Delta = \frac{\Delta_1 + \Delta_2 + \Delta_3}{1 - \Delta_3}$ . In view of  $\Delta_1 + \Delta_2 + \Delta_3 < 1$ , then  $0 \leq \Delta < 1$ . Thus,  $\{\alpha_n\}$  is a  $\Delta$ -Cauchy sequence in  $\mathcal{Y}$ . Hence  $(\mathcal{Y}, \vartheta)$  is  $\Delta$ -complete, there exists a point  $u^* \in \mathcal{Y}$  such that  $\alpha_n \rightarrow u^*$  as  $n \rightarrow \infty$ .

By (3.6), it's easy to see that

$$\begin{aligned} \vartheta(u^*, Tu^*) &\leq s\{\vartheta(u^*, \alpha_{n+1}) + \vartheta(\alpha_{n+1}, Tu^*)\} \\ &= \{(u^*, \alpha_{n+1})\} + \{(T\alpha_n, Tu^*)\} \\ &\leq \{(u^*, \alpha_{n+1})\} + s\{\Delta_1(\alpha_n, u^*) + \\ &\quad \Delta_2 \frac{\vartheta(\alpha_n, T\alpha_n)\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, Tu^*)\vartheta(u^*, T\alpha_n)}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, T\alpha_n)} + \Delta_3 [\vartheta(\alpha_n, T\alpha_n) + \vartheta(u^*, Tu^*)]\} \\ &\leq s\{\vartheta(u^*, \alpha_{n+1})\} + s\{\Delta_1 \vartheta(\alpha_n, u^*) + \\ &\quad \Delta_2 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, Tu^*)\vartheta(u^*, \alpha_{n+1})}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, \alpha_{n+1})} + \\ &\quad \Delta_3 [\vartheta(\alpha_n, \alpha_{n+1})\vartheta(u^*, Tu^*)]\} \quad (3.9) \end{aligned}$$

Taking limit as  $n \rightarrow \infty$  by both components of (3.9), we have

$$\lim_{n \rightarrow \infty} \vartheta(u^*, Tu^*) = 0.$$

Then,

$Tu^* = u^*$  and  $u^*$  is a fixed point of T. In the end, we have to prove the uniqueness of the fixed point. Indeed, if there is

Another fixed point  $v^*$ , then by (3.6),  $\vartheta(u^*, v^*) = \vartheta(Tu^*, Tv^*)$

$$\begin{aligned} &\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, Tu^*)\vartheta(u^*, Tv^*) + \vartheta(v^*, Tv^*)\vartheta(v^*, Tu^*)}{\vartheta(u^*, Tv^*) + \vartheta(v^*, Tu^*)} + \\ &\quad \Delta_3 [\vartheta(u^*, Tu^*) + \vartheta(v^*, Tv^*)] \\ &\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, u^*)\vartheta(u^*, v^*) + \vartheta(v^*, v^*)\vartheta(v^*, u^*)}{\vartheta(u^*, v^*) + \vartheta(v^*, u^*)} + \\ &\quad \Delta_3 [\vartheta(u^*, u^*) + \vartheta(v^*, v^*)] \\ (u^*, v^*) &\leq \Delta_1 (u^*, v^*) \quad (3.10) \end{aligned}$$

Since  $\Delta_1 + \Delta_2 + \Delta_3 < 1$  implies  $\Delta_1 + \Delta_3 < 1$ , we have  $\vartheta(u^*, v^*) = 0$ , i.e.,  $u^* = v^*$ .

**Theorem-3.3**

Consider  $\mathcal{Y}$  be a  $\mathcal{b}$ -complete  $\mathcal{b}$ -metric space. It has coefficient  $s \geq 1$  and  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  be a mapping such that

$$\vartheta(T\alpha, T\beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\alpha, T\beta) + \vartheta(\beta, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} + \Delta_3 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)} + \Delta_4 \frac{\vartheta(\alpha, T\beta)\vartheta(\beta, T\alpha)}{\vartheta(\alpha, \beta)} + \Delta_5 [\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)]$$

for all  $\alpha, \beta \in \mathcal{Y}$  and  $\Delta_1, \Delta_2, \Delta_3, \Delta_4$  and  $\Delta_5$  are nonnegative constants with  $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + 2\Delta_5 < 1$ . Hence  $T$  has a unique fixed point in  $\mathcal{Y}$ .

*Proof:*

Let  $\alpha_0$  be arbitrary in  $\mathcal{Y}$ , we define a sequence  $\{\alpha_n\}$  in  $\mathcal{Y}$  such that  $\alpha_{n+1} = T\alpha_n$ ,  $\alpha_n = T\alpha_{n-1}$ , for all  $n \in \mathbb{N}$ , from the condition (3.11) with  $\alpha = \alpha_{n-1}$  and  $\beta = \alpha_n$ . Therefore

$$\begin{aligned} (\alpha_n, \alpha_{n+1}) &= (T\alpha_{n-1}, T\alpha_n) \\ &\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, T\alpha_{n-1})\vartheta(\alpha_{n-1}, T\alpha_n) + \vartheta(\alpha_n, T\alpha_n)\vartheta(\alpha_n, T\alpha_{n-1})}{\vartheta(\alpha_{n-1}, T\alpha_n) + \vartheta(\alpha_n, T\alpha_{n-1})} + \\ &\Delta_3 \frac{\vartheta(\alpha_{n-1}, T\alpha_{n-1})\vartheta(\alpha_n, T\alpha_n)}{\vartheta(\alpha_{n-1}, \alpha_n)} + \Delta_4 \frac{\vartheta(\alpha_{n-1}, T\alpha_n)\vartheta(\alpha_n, T\alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_n)} + \end{aligned}$$

$$\begin{aligned} &\Delta_5 [\vartheta(\alpha_{n-1}, T\alpha_{n-1}) + \vartheta(\alpha_n, T\alpha_n)] \\ &\leq \Delta_1 \vartheta(\alpha_{n-1}, \alpha_n) + \Delta_2 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_{n-1}, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_n, \alpha_n)}{\vartheta(\alpha_{n-1}, \alpha_{n+1}) + \vartheta(\alpha_n, \alpha_n)} + \\ &\Delta_3 \frac{\vartheta(\alpha_{n-1}, \alpha_n)\vartheta(\alpha_n, \alpha_{n+1})}{\vartheta(\alpha_{n-1}, \alpha_n)} + \Delta_4 \frac{\vartheta(\alpha_{n-1}, \alpha_{n+1})\vartheta(\alpha_n, \alpha_n)}{\vartheta(\alpha_{n-1}, \alpha_n)} + \\ &\Delta_5 [\vartheta(\alpha_{n-1}, \alpha_n) + \vartheta(\alpha_n, \alpha_{n+1})] \\ &\leq \Delta_1 (\alpha_{n-1}, \alpha_n) + \Delta_2 (\alpha_{n-1}, \alpha_n) + \Delta_3 (\alpha_n, \alpha_{n+1}) + \Delta_4 (0) + \Delta_5 (\alpha_{n-1}, \alpha_n) + \Delta_5 (\alpha_n, \alpha_{n+1}) \end{aligned}$$

It follows that

$$\begin{aligned} \vartheta(\alpha_n, \alpha_{n+1})(1 - \Delta_3 - \Delta_5) &\leq (\Delta_1 + \Delta_2 + \Delta_5)\vartheta(\alpha_{n-1}, \alpha_n) \\ \vartheta(\alpha_n, \alpha_{n+1}) &\leq \frac{\Delta_1 + \Delta_2 + \Delta_5}{1 - \Delta_3 - \Delta_5} \vartheta(\alpha_{n-1}, \alpha_n) \end{aligned} \tag{3.12}$$

$$\frac{\Delta_1 + \Delta_2 + \Delta_5}{1 - \Delta_3 - \Delta_5}$$

Put  $1 - \Delta_3 - \Delta_5$ . In view of  $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + 2\Delta_5 < 1$ , then  $0 \leq \Delta \leq 1$ . Thus,  $\{\alpha_n\}$  is a  $\mathcal{b}$ -Cauchy sequence in  $\mathcal{Y}$ . Since  $(\mathcal{Y}, \vartheta)$  is  $\mathcal{b}$ -Complete, there exists a point  $u^* \in \mathcal{Y}$  thus  $\alpha_n \rightarrow u^*$  as  $n \rightarrow \infty$ .

By (3.11), it's easy to see that

$$\begin{aligned} \vartheta(u^*, Tu^*) &\leq s\{\vartheta(u^*, \alpha_{n+1}) + \vartheta(\alpha_{n+1}, Tu^*)\} \\ &= \{(u^*, \alpha_{n+1}) + (T\alpha_n, Tu^*)\} \\ &\leq \{(u^*, \alpha_{n+1})\} + s\Delta_1(\alpha_n, u^*) + \\ &s\Delta_2 \frac{\vartheta(\alpha_n, T\alpha_n)\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, Tu^*)\vartheta(u^*, T\alpha_n)}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, T\alpha_n)} + s\Delta_3 \frac{\vartheta(\alpha_n, T\alpha_n)\vartheta(u^*, Tu^*)}{\vartheta(\alpha_n, u^*)} + \\ &s\Delta_4 \frac{\vartheta(\alpha_n, Tu^*)\vartheta(u^*, T\alpha_n)}{\vartheta(\alpha_n, u^*)} + s\Delta_5 [\vartheta(\alpha_n, T\alpha_n) + \vartheta(u^*, Tu^*)] \\ &\leq \{(u^*, \alpha_{n+1})\} + s\Delta_1(\alpha_n, u^*) + \\ &s\Delta_2 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, Tu^*)\vartheta(u^*, \alpha_{n+1})}{\vartheta(\alpha_n, Tu^*) + \vartheta(u^*, T\alpha_n)} + s\Delta_3 \frac{\vartheta(\alpha_n, \alpha_{n+1})\vartheta(u^*, Tu^*)}{\vartheta(\alpha_n, u^*)} + \\ &s\Delta_4 \frac{\vartheta(\alpha_n, Tu^*)\vartheta(u^*, \alpha_{n+1})}{\vartheta(\alpha_n, u^*)} + s\Delta_5 [\vartheta(\alpha_n, \alpha_{n+1}) + \vartheta(u^*, Tu^*)] \end{aligned}$$

Taking the limit as  $n \rightarrow \infty$  by both components of (3.13), we have  $\lim_{n \rightarrow \infty} \vartheta(u^*, Tu^*) = 0$ .

Then,  $Tu^* = u^*$  and  $u^*$  is a fixed point of  $T$ .

In the end, we prove the uniqueness of the fixed point. Indeed, if there is another fixed point  $v^*$ , then by (3.11),  $\vartheta(u^*, v^*) = \vartheta(Tu^*, Tv^*)$

$$\begin{aligned} &\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, Tu^*)\vartheta(u^*, Tv^*) + \vartheta(v^*, Tv^*)\vartheta(v^*, Tu^*)}{\vartheta(u^*, Tv^*) + \vartheta(v^*, Tu^*)} + \\ &\Delta_3 \frac{\vartheta(u^*, Tu^*)\vartheta(v^*, Tv^*)}{\vartheta(u^*, v^*)} + \Delta_4 \frac{\vartheta(u^*, Tv^*)\vartheta(v^*, Tu^*)}{\vartheta(u^*, v^*)} + \Delta_5 [\vartheta(u^*, Tu^*) + \vartheta(v^*, Tv^*)] \\ &\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_2 \frac{\vartheta(u^*, u^*)\vartheta(v^*, v^*) + \vartheta(v^*, v^*)\vartheta(v^*, u^*)}{\vartheta(u^*, v^*) + \vartheta(v^*, u^*)} + \end{aligned}$$

$$\Delta_3 \frac{\vartheta(u^*, u^*)\vartheta(v^*, v^*)}{\vartheta(u^*, v^*)} + \Delta_4 \frac{\vartheta(u^*, v^*)\vartheta(v^*, u^*)}{\vartheta(u^*, v^*)} + \Delta_5 [\vartheta(u^*, u^*) + \vartheta(v^*, v^*)]$$



$$\begin{aligned} &\leq \Delta_1 \vartheta(u^*, v^*) + \Delta_4 \vartheta(v^*, u^*) \\ &\leq (\Delta_1 + \Delta_4) [\vartheta(u^*, v^*)] \end{aligned} \tag{3.14}$$

Since  $0 < \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + 2\Delta_5 < 1$  implies  $\Delta_1 + \Delta_3 + 2\Delta_5 < 1$ , then we have  $(u^*, v^*) = 0$ . Thus, we proved that T have a unique fixed point in  $\mathcal{Y}$ .

### Application for Non-Linear Integral Equations

Assume that,  $\mathcal{Y} = \mathcal{A}[u, v]$  is the set of all real valued continuous functions on  $[u, v]$ , that  $[u, v]$  is the closed and bounded interval in  $\mathbb{R}$ . For  $\zeta > 1$  is a real number, define  $\vartheta: \mathcal{Y} \times \mathcal{Y} \rightarrow \mathbb{R}_+$  by:  $(\alpha, \beta) = \sup_{\lambda \in [u, v]} |\alpha(\lambda) - \beta(\lambda)|^\zeta$  For all  $\alpha, \beta \in \mathcal{Y}$ . Therefore,  $(\mathcal{Y}, \vartheta)$  is a  $b$ -complete  $b$ -metric space with  $\mu = 2^{\zeta-1}$ . Here we apply Theorem 2.10 to establish the existence of solution of nonlinear integral equation of Fredholm type and it is defined by:

$$\alpha(\lambda) = \eta(\lambda) + \Delta \int_u^v \mathcal{H}(\lambda, \mu, \alpha(\mu)) d\mu \tag{3.15}$$

Where  $\alpha \in \mathcal{A}[u, v]$  is the unknown function,  $\Delta \in \mathbb{R}$ ,  $\lambda, \mu \in [u, v]$ ,  $\mathcal{H}: [u, v] \times [u, v] \times \mathbb{R} \rightarrow \mathbb{R}$  and  $\eta: [u, v] \rightarrow \mathbb{R}$  are continuous functions.

**Theorem 3.4.** Let the following conditions are satisfied.

1. There exists a continuous function  $\tau: [u, v] \times [u, v] \rightarrow \mathbb{R}_+$  thus for all  $\alpha, \beta \in \mathcal{Y}$ ,  $\Delta \in \mathbb{R}$  and  $\lambda, \mu \in [u, v]$ , we have  $\zeta |\mathcal{H}(\lambda, \mu, \alpha(\mu)) - \mathcal{H}(\lambda, \mu, \beta(\mu))|$

Where

$$\mathcal{M}(\alpha, \beta) \leq \Delta_1 \vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\alpha) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\beta)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} + \Delta_3 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)}$$

2.  $\Delta \leq 1$ .

$$\sup_{\lambda \in [u, v]} \int_u^v \tau(\lambda, \mu) d\mu \leq \frac{1}{2^{\zeta-1}(v-u)^{\zeta-1}}$$

Then, the nonlinear integral equation (3.15) has a solution  $z \in \mathcal{A}[u, v]$ .

*Proof:*

Construct the mapping  $T: \mathcal{Y} \rightarrow \mathcal{Y}$  by:

$$T\alpha(t) = \eta(t) + \Delta \int_u^v \mathcal{H}(\lambda, \mu, \alpha(\mu)) d\mu$$

For all  $\lambda \in [u, v]$ . So, the existence of the solution of a (3.15) is equivalent to the existence and uniqueness of fixed point of  $T$ .

Let  $\rho \in \mathbb{R}$  such that  $\frac{1}{\zeta} + \frac{1}{\rho} = 1$ . Applying the Holder inequality, (1), (2) and (3), we have  $(T\alpha, T\beta) =$

$$\begin{aligned} &\leq |\Delta|^\zeta \sup_{\lambda \in [u, v]} \left( \int_u^v |\mathcal{H}(\lambda, \mu, \alpha(\mu)) - \mathcal{H}(\lambda, \mu, \beta(\mu))| d\mu \right)^\zeta \\ &\leq \sup_{\lambda \in [u, v]} \left[ \int_u^v 1^\rho d\mu^{\frac{1}{\rho}} \left( \int_u^v |\mathcal{H}(\lambda, \mu, \alpha(\mu)) - \mathcal{H}(\lambda, \mu, \beta(\mu))|^\zeta d\mu \right)^{\frac{1}{\zeta}} \right]^\zeta \end{aligned}$$

$$\begin{aligned} &\leq (v - u)^{\zeta-1} \sup_{\lambda \in [u, v]} \left( \int_u^v \tau(\lambda, \mu) d\mu \mathcal{M}(\alpha, \beta) \right) \\ &\leq (v - u)^{\zeta-1} \sup_{\lambda \in [u, v]} \left( \int_u^v \tau(\lambda, \mu) d\mu \right) \mathcal{M}(\alpha, \beta) \\ &\leq \frac{1}{2^{\zeta-1}} \mathcal{M}(\alpha, \beta) \end{aligned}$$

Thus

$$\mu\vartheta(T\alpha, T\beta) \leq \Delta_1\vartheta(\alpha, \beta) + \Delta_2 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\alpha) + \vartheta(\beta, T\beta)\vartheta(\alpha, T\beta)}{\vartheta(\alpha, T\beta) + \vartheta(\beta, T\alpha)} + \Delta_3 \frac{\vartheta(\alpha, T\alpha)\vartheta(\beta, T\beta)}{\vartheta(\alpha, \beta)}$$

Thus the all conditions of Theorem 2.10 holds. Consequently, the integral equation (3.15) has a solution  $z \in \mathcal{A}[u, v]$ .

## CONCLUSION

In this project, basic concepts of *m*-metric space and fixed point theorems are demonstrated, with some instances providing verification. Next, we have defined a new class of functions for defined some new contractive mappings in *b-m*-metric spaces.

Furthermore, we proved some fixed point results for these contractive mappings. Afterwards we prove the Application to Non-Linear Integral Equations by one theorem.

In summary, the study of fixed point results in rational type contraction mappings within *b-m*-metric spaces not only enriches the theoretical landscape of mathematics but also offers practical tools for addressing challenging problems in science, engineering and beyond.

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**AN OPTMIZATION OF INVENTORY MODEL USING ECONOMIC LOT SIZE AND FINITE RATE DELIVERY METHODS WITH CONSIDERATION OF DEFECTIVE PRODUCTS**

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**ABSTRACT**

Inventory management plays a vital role in balancing supply and demand while minimizing costs. Determining lot order quantities involves weighing various factors, including inventory holding costs, demand variability, and production constraints. However, two critical yet often overlooked factors are goodwill penalty cost and reworking cost. Goodwill Penalty Cost emerges from stock outs or delayed deliveries, impacting customer satisfaction and brand reputation. It encompasses intangible costs such as customer dissatisfaction, potential loss of future sales, and harm to brand equity. Reworking Cost, on the other hand, stems from correcting defects in inventory, encompassing expenses related to identifying, rectifying, or discarding faulty products. This abstract explores the significance of integrating these costs into lot order determination models. It highlights the necessity of striking a balance between minimizing inventory holding costs and avoiding potential losses due to quality issues. It emphasizes the need for robust inventory models, including EOQ, EPQ, or advanced techniques like demand forecasting and risk assessment, to optimize lot order quantities while considering goodwill penalty cost and reworking cost. By incorporating these critical

considerations into lot order determination, businesses can enhance customer satisfaction, safeguard brand reputation, optimize production efficiency, and minimize waste. This abstract aims to underscore the importance of considering these costs in inventory management strategies to achieve a holistic approach to cost optimization and customer fulfilment.

**Keywords:** Economic Order Quantity, Economic Production Quantity, Robust Inventory model, Optimizing Inventory.

## I. INTRODUCTION

Inventory is a stock of tangible assets with monetary, material, or labour worth. Raw materials, tools and accessories, including spare components required in production, unfinished or in-process inventories, and final goods are examples of material inventory. Inventories are kept at a cost in order to achieve monetary benefits, such as preventing a shutdown due to a temporary lack of supplies or allowing uniform production for a suitable supply or demand. Inventory management is a critical facet of business operations aimed at balancing supply and demand while optimizing costs. Central to this is the determination of lot order quantities, where considerations like goodwill penalty cost and reworking cost play pivotal roles. Goodwill penalty cost arises due to insufficient inventory leading to stockouts or delays in fulfilling customer orders. When a company fails to meet customer demands promptly, it incurs costs beyond the immediate financial impact, such as damage to reputation, loss of customer loyalty, and potential future sales.

Calculating goodwill penalty cost involves estimating the adverse effects on customer relationships and brand reputation due to stockouts or delayed deliveries.

Minimizing this cost is crucial to maintaining customer satisfaction and preserving brand value. Reworking cost is incurred when defective or substandard inventory requires correction or reprocessing before it can be sold or utilized in production. This cost includes expenses associated with identifying, fixing, and potentially scrapping or reworking defective items. Determining reworking cost involves assessing the direct expenses of reprocessing, potential labor costs, additional material requirements, and the impact on production schedules. Minimizing reworking cost is essential for maintaining product quality, reducing waste, and optimizing operational efficiency.

## II. LITERATURE REVIEW

In the realm of research on the inventory lot-sizing concept, particularly when involving reworking and remanufacturing processes, Chiu et al. (2010) introduced a straightforward algebraic approach to determine the optimal lot size for the EPQ model with a rework process. Addressing imperfect reworking of substandard quality items, Akbarzadeh et al. (2016) discussed an EPQ model. Taleizadeh et al. (2016) designed an EPQ inventory model considering reworkable defective items within the context of a given multi-shipment policy. Exploring the lot-sizing problem in a hybrid manufacturing and remanufacturing system with a one-way substitution option, Zouadi et al. (2019) conducted an investigation. Attila et al. (2020) delved into a lot-sizing problem involving the remanufacturing option under parameter uncertainties related to demands and returns. Recent works have shown a focus on the lot-sizing concept, enriched by incorporating shortage and partial backorder terms. Abad (2008) addressed the lot-sizing problem by considering the general rate of deterioration and partial backordering. Cárdenas-Barrón

(2009) developed an EPQ inventory model with planned backorders to determine the EPQ for a single product. Additionally, he expanded the provided model by Jamal et al. (2004) by incorporating planned backorders. Drake et al. (2011) tackled a lot-sizing problem for a two-stage system where the final product was planned using an EPQ model with partial backordering. Cárdenas-Barrón (2011) proposed a straightforward method integrating analytic, geometric, and algebraic approaches to determine optimal lot sizes and backorder levels, incorporating linear and fixed backorder costs. Taleizadeh et al. (2011) investigated multiproduct joint production systems, exploring scenarios with and without reworking processes and treating shortages as backorders. Pentico and Drake (2009) formulated equations for the EOQ model, considering partial backorders and later extended their work to address an increased percentage of backordered demands when production recommenced (Pentico et al., 2011). Toews et al. (2011) further extended models to accommodate a constant backordering rate increasing linearly over time. Building upon the work of Pentico et al. (2011), Wee and Wang (2012) assumed an increase in backordering rate only from the start of the production phase, presenting an alternative approach for solving a lot-sizing model with backorders using Cauchy-Bunyakovsky-Schwarz Inequality (Wee et al., 2013). Pentico et al. (2014) designed two heuristics for basic EOQ and EPQ models with partial backordering, employing time between orders and the percentage of demand filled from stock as decision variables. Skouri et al. (2014) examined a single-echelon inventory installation under the classical EOQ paradigm with backorders. In exploring diverse payment structures, Zia et al. developed a hybrid model incorporating linked-to-order multiple advance payments and delayed payments in a lot-sizing model, permitting backordering shortages. Sadeghi et

al. (2016) introduced a multi-item EPQ model with fuzzy demand where shortages were backordered under warehouse space constraints. Hsu and Hsu (2016) formulated an EPQ model to determine optimal lot sizes and backorder quantities for a manufacturer under an imperfect production process. Taleizadeh (2017) developed a lot-sizing model, considering partial backorders in cases of shortages. Pacheco-Velázquez and Cárdenas-Barrón (2016) considered ordering and holding costs for raw materials and finished products in a lot-sizing EPQ model.

### III. NOTATIONS AND ASSUMPTIONS

The integrated model employs the notations and assumptions listed below.

#### Notations:

- $C_S$  - Setup cost of an item per period
- $C_H$  - Holding cost of an item per period
- $C_C$  - Cycle cost of an item
- $C_P$  - Cost per period
- $C_R$  - Unit return cost of an item
- $C_G$  - Unit penalty cost from goodwill loss
- $C_W$  - Cost of reworking of an item
- $E = L$  - Maximum Inventory, where L is the lot order
- V - Rate of use of an item per period
- B - Safety stock
- P - Item per unit of time is a manufacturing rate
- K - Demand
- $\beta$  - Percentage of imperfect items
- $x$  - Percentage of defective items per cycle

$y$  - Percentage of scrap in defective items per cycle

ROL - Reorder level

$t$  - Unit of time from zero time lasting  $dt$  units of time

$t_l$  - Lead time

$n$  - Duration of cycle of an inventory

$m$  - Duration of positive inventory

**Assumptions:**

- i. Delivery of an item occurs instantaneously when the stock hits zero. Order placement and delivery charges are fixed and independent by the size of the lot.
- ii. The rate of usage is constant.
- iii. There is no safety stock offered.
- iv. There can be no shortage.
- v. The quantity of goods stored in the inventory at any given time determines the warehousing cost, which will be determined continuously.
- vi. Products that are defective are reworked and have minor damage that can be repaired in a controlled manner.
- vii. The percentage of defective products is provided and is well-known.

**IV. MATHEMATICAL MODEL**

**Economic Lot size:**

At any given time, the inventory is a straight line function of time and is  $L$  at  $t = 0$  and  $0$  at  $t = n$ , where  $n$  is the duration of the cycle.

$$\text{Inventory} = \frac{-L}{n}t + L$$

The number of periods of time  $n$  that the lot  $L$  will last given by,

$$n = \frac{\text{items}}{\text{items/period}}$$

$$n = \frac{L}{V}$$

$$\text{Inventory} = -Vt + L$$

For the defective products,

$$\begin{aligned} &\text{Total reworking cost of the product} \\ &= \frac{C_W K m (1-y)}{(1-xy)} \end{aligned}$$

$$\text{Total goodwill penalty cost of the product} = (C_R + C_G)\beta K m$$

The total cycle cost is given by,

$$\begin{aligned} C_C &= C_S + \int_0^{n=\frac{L}{V}} C_H (-Vt + L) dt + \\ &(C_R + C_G)\beta K m + \frac{C_W K m (1-y)}{(1-xy)} \\ &= C_S + C_H \int_0^{n=\frac{L}{V}} (-Vt + L) dt + \\ &(C_R + C_G)\beta K m + \frac{C_W K m (1-y)}{(1-xy)} \\ &= C_S + C_H \left[ \frac{-L^2}{2V} + \frac{L^2}{V} \right] + \\ &K m \left[ (C_R + C_G)\beta + \frac{C_W (1-y)}{(1-xy)} \right] \\ &= C_S + C_H \left[ \frac{L^2}{2V} \right] + K m \left[ (C_R + \right. \\ &\left. C_G)\beta + \frac{C_W (1-y)}{(1-xy)} \right] \end{aligned}$$

Here cost per period  $C_P$  is the function that has to be optimized.

$$\text{Cycle per period} = \frac{\text{item/period}}{\text{item/cycle}} = \frac{V}{L}$$

Cost per period = Cycle cost  $\times$  Cycle per period

$$\begin{aligned} C_P &= \left[ C_S + C_H \left[ \frac{L^2}{2V} \right] + K m \left[ (C_R + \right. \right. \\ &\left. \left. C_G)\beta + \frac{C_W (1-y)}{(1-xy)} \right] \right] \times \left[ \frac{V}{L} \right] \\ &= \frac{C_S V}{L} + \frac{C_H L}{2} + \frac{K m V}{L} \left[ (C_R + C_G)\beta + \right. \\ &\left. \frac{C_W (1-y)}{(1-xy)} \right] \end{aligned}$$

$$= \frac{C_H L}{2} + \frac{V}{L} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]$$

Differentiating  $C_P$  and equating to 0,

$$\frac{dC_P}{dL} = \frac{C_H}{2} - \frac{V}{L^2} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]$$

$$\frac{C_H}{2} - \frac{V}{L^2} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right] = 0$$

$$\frac{C_H}{2} = \frac{V}{L^2} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]$$

$$L^2 = \frac{2V}{C_H} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]$$

$$L_{opt} = \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]}$$

Substituting  $L_{opt}$  in cost per period  $C_P, C_P =$

$$\frac{C_H}{2} \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]} +$$

$$\frac{V}{\sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right}}} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]$$

$$C_P = \sqrt{\frac{C_H V}{2} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]} + \sqrt{\frac{C_H V}{2} \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]}$$

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \right]}$$

Here, Reorder level = Lead time  $\times$  Rate of use per item

$$ROL = t_l \times V$$

### Safety Stock

This is added to the given inventory to make sure that there is no shortage of an item. Hence the inventory model becomes,

$$\therefore \text{Inventory} = -Vt + L + B$$

The total cycle cost is given by,

$$C_C = C_S + \int_0^{n=\frac{L}{V}} C_H (-Vt + L + B) dt + (C_R + C_G) \beta Km + \frac{C_W Km(1-y)}{(1-xy)}$$

$$= C_S + C_H \int_0^{n=\frac{L}{V}} (-Vt + L + B) dt + (C_R + C_G) \beta Km + \frac{C_W Km(1-y)}{(1-xy)}$$

$$= C_S + C_H \left[ \frac{-L^2}{2V} + \frac{L^2}{V} + \frac{BL}{V} \right] + Km \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right]$$

$$= C_S + C_H \left[ \frac{L^2}{2V} + \frac{BL}{V} \right] + Km \left[ (C_R + C_G) \beta + \frac{C_W(1-y)}{(1-xy)} \right]$$

Cost per period = Cycle cost  $\times$  Cycle per period

$$C_P = \left[ C_S + C_H \left[ \frac{L^2}{2V} + \frac{BL}{V} \right] + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] \times \left[ \frac{V}{L} \right]$$

$$= \frac{C_S V}{L} + C_H \left[ \frac{L}{2} + B \right] + \frac{KmV}{L} \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right]$$

$$C_P = \frac{C_H L}{2} + \frac{C_S V}{L} + C_H B + \frac{KmV}{L} \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right]$$

$$= \frac{C_H L}{2} + C_H B + \frac{V}{L} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

Differentiating  $C_P$  and equating to 0,

$$\frac{dC_P}{dL} = \frac{C_H}{2} - \frac{V}{L^2} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

$$\frac{C_H}{2} - \frac{V}{L^2} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] = 0$$

$$\frac{C_H}{2} = \frac{V}{L^2} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

$$L^2 = \frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

$$L_{opt} = \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

Substituting  $L_{opt}$  in cost per period  $C_P$ ,

$$C_P = \frac{C_H}{2} \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]} + C_H B + \frac{V}{\sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right}}} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

$$C_P = \sqrt{\frac{C_H V}{2} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]} + C_H B + \sqrt{\frac{C_H V}{2} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]} + C_H B$$

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

when  $B = 0$



## Finite Rate Delivery

This model is associated with manufacturing, the value for  $L$  is the total number of items produced and used in the cycle of  $(t_1 + t_2)$  periods.

In time  $t_1$  the inventory builds up by an amount  $(E - B)$  at a rate  $(P - V)$ .

Thus,  $(E - B) = t_1(P - V)$  and  $L = t_1P$  (Since  $L$  is made at  $P$  rate in time  $t_1$ ).

Eliminating  $t_1$  gives,  $(E - B) = L \frac{(P-V)}{P}$

But  $L = Vn$  ( $L$  is used in  $n$  period at  $V$  rate)

$$(E - B) = Vn \frac{(P-V)}{P}$$

In this model,  $(E - B)$  is linear with time on both sides of the maximum inventory.

$$\text{Now, Average Inventory} = \frac{Vn(P-V)}{2} B$$

The Average Inventory is held at a cost  $C_H$  per item per period.

The Cost per period  $C_P$  is,

$$\begin{aligned} C_P &= \left[ C_S + \left[ \frac{Vn(P-V)}{2} + B \right] C_H n \right] + \\ & (C_R + C_G) \beta K m + \frac{C_W K m (1-y)}{(1-xy)} \times \frac{1}{n} \\ &= \frac{C_S}{n} + \frac{C_H V (P-V)}{2} n + C_H B + \\ & \frac{K m}{n} \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \\ &= \frac{C_H V (P-V)}{2} n + C_H B + \\ & \frac{1}{n} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right] \end{aligned}$$

Differentiating  $C_P$  and equating to 0,

$$\frac{dC_P}{dn} = \frac{C_H V (P-V)}{2} - \frac{1}{n^2} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]$$

$$\frac{C_H V (P-V)}{2} - \frac{1}{n^2} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right] = 0$$

$$\frac{C_H V (P-V)}{2} = \frac{1}{n^2} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]$$

$$n^2 = \frac{2P}{C_H V (P-V)} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]$$

$$n_{opt} = \sqrt{\frac{2P}{C_H V (P-V)} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]}$$

$$\frac{L}{V} = \sqrt{\frac{2P}{C_H V (P-V)} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]}$$

$\therefore n = \frac{L}{V}$

$$L_{opt} = \sqrt{\frac{2PV}{C_H (P-V)} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]}$$

Substituting  $n_{opt}$  in cost per period  $C_P$ ,

$$\begin{aligned} C_P &= \\ & \frac{C_H V (P-V)}{2} \frac{L}{V} + \frac{L}{V} \sqrt{\frac{2P}{C_H V (P-V)} \left[ C_S + K m \left[ (C_R + C_G) \beta + \frac{C_W (1-y)}{(1-xy)} \right] \right]} + \\ & C_H B + \end{aligned}$$

$$\frac{1}{\sqrt{\frac{2P}{C_H V(P-V)} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]$$

$$C_P = \sqrt{\frac{2C_H V(P-V)}{P} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right] + C_H}$$

### Optimal Solution:

#### 1) Economic Lot Size

##### Optimum Lot Order

$$L_{opt} = \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

##### Optimum Period Cost

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

##### Finite Rate Delivery

**Optimum Lot Order**  $L_{opt} =$

$$\sqrt{\frac{2PV}{C_H(P-V)} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

## V. NUMERICAL EXAMPLES

### Example 1:

Consider an inventory model with the following parameters:  $C_S = 0.84$ ,  $C_H = 0.05$ ,

$C_R = 0.9$ ,  $C_G = 0.96$ ,  $C_W = 0.36$ ,  $V = 150$ ,  
 $P = 300$ ,  $B = 40$ ,  $K = 300$ ,  $m = 5$ ,  $\beta = 2$ ,  
 $x = 0.045$ ,  $y = 0.6$

### Economic Lot Size

#### (i) Optimum Lot Order

$$L_{opt} = \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

$$L_{opt} = \sqrt{\frac{2(150)}{0.05} \left[ 0.84 + ((300)(5)) \left[ (0.9 + 0.96)2 + \frac{0.36(1-0.6)}{(1-((0.045)(0.6)))} \right] \right]}$$

$$L_{opt} = 5895.71 \approx 5896$$

#### (ii) Optimum Period Cost

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$

$$C_P = \sqrt{2(0.05)(150) \left[ 0.84 + ((300)(5)) \left[ (0.9 + 0.96)2 + \frac{0.36(1-0.6)}{(1-((0.045)(0.6)))} \right] \right]}$$

$$C_P = 294.7 \approx 295/\text{day}$$

### Finite Rate Delivery

#### (i) Optimum Lot Order

$$L_{opt} = \sqrt{\frac{2(300)(150)}{(0.05)(300-150)} \left[ 0.84 + ((300)(5)) \left[ (0.9 + 0.96)2 + \frac{0.36(1-0.6)}{(1-((0.045)(0.6)))} \right] \right]}$$

$$L_{opt} = 8337.7 \approx 8338$$

#### (ii) Optimum Period Cost

$$C_P = \sqrt{\frac{2C_H V(P-V)}{P} \left[ 0.84 + ((300)(5)) \left[ (0.9 + 0.96)2 + \frac{0.36(1-0.6)}{(1-((0.045)(0.6)))} \right] \right] + ((0.05)(40))}$$

$$C_P = \sqrt{\frac{2(0.05)(150)(300-150)}{300} \left[ 0.84 + 1500 \left[ (1.86)2 + \frac{0.36(0.4)}{1-0.0271} \right] \right] + 2}$$

$$C_P = 210.4 \approx 210/\text{day}$$

### Example 2:

Consider an inventory model with the following parameters:  $C_S = 0.67$ ,  $C_H = 0.04$ ,  $C_R = 0.7$ ,  $C_G = 0.8$ ,  $C_W = 0.2$ ,  $V = 100$ ,  $P = 200$ ,  $B = 25$ ,  $K = 200$ ,  $m = 3$ ,  $\beta = 1.5$ ,  $x = 0.025$ ,  $y = 0.4$

### Economic Lot Size

#### (i) Optimum Lot Order

$$L_{opt} = \sqrt{\frac{2V}{C_H} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$
$$L_{opt} = \sqrt{\frac{2(100)}{0.04} \left[ 0.67 + ((200)(3)) \left[ (0.7 + 0.8)1.5 + \frac{0.2(1-0.4)}{1 - ((0.025)(0.4))} \right] \right]}$$

$$L_{opt} = 2667.9 \approx 2668$$

#### (ii) Optimum Period Cost

$$C_P = \sqrt{2C_H V \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$
$$C_P = \sqrt{2(0.04)(100) \left[ 0.67 + ((200)(3)) \left[ (0.7 + 0.8)1.5 + \frac{0.2(1-0.4)}{1 - ((0.025)(0.4))} \right] \right]}$$

$$C_P = 106.7 \approx 107/day$$

### Finite Rate Delivery

#### (i) Optimum Lot Order

$$L_{opt} = \sqrt{\frac{2PV}{C_H(P-V)} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]}$$
$$L_{opt} = \sqrt{\frac{2(200)(100)}{0.04(200-100)} \left[ 0.67 + ((200)(3)) \left[ (0.7 + 0.8)1.5 + \frac{0.2(1-0.4)}{1 - ((0.025)(0.4))} \right] \right]}$$

$$L_{opt} = 3773.03 \approx 3773$$

#### (ii) Optimum Period Cost

$$C_P = \sqrt{\frac{2C_H V (P-V)}{P} \left[ C_S + Km \left[ (C_R + C_G)\beta + \frac{C_W(1-y)}{(1-xy)} \right] \right]} + C_H B$$
$$C_P = \sqrt{\frac{2(0.04)(100)(200-100)}{200} \left[ 0.67 + ((200)(3)) \left[ (0.7 + 0.8)1.5 + \frac{0.2(1-0.4)}{1 - ((0.025)(0.4))} \right] \right]} + (0.04)(25)}$$

$$C_P = 76.46 \approx 76/day$$

## VI. CONCLUSION

Integrating goodwill penalty cost and reworking cost considerations into lot order determination involves a delicate balance between minimizing inventory holding costs and mitigating potential losses from stockouts, delays, or quality issues. Optimizing lot order quantities requires a thorough analysis that accounts for demand variability, lead times, economies of scale, carrying costs, and the costs associated with goodwill penalties and reworking. Implementing effective inventory management strategies that consider goodwill penalty cost and reworking cost in lot order determination can lead to enhanced customer satisfaction, improved brand reputation, reduced waste, and increased operational efficiency. This necessitates the adoption of robust inventory models, such as Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), or newer techniques like demand forecasting, risk assessment, and simulation to strike a balance between cost optimization and meeting customer expectations.

In conclusion, the integration of goodwill penalty cost and reworking cost considerations in determining lot order quantities is imperative for businesses aiming to achieve efficient inventory management, ensuring timely customer fulfillment, maintaining product quality, and optimizing overall operational costs.

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### MEMORIES: A TRIGGER FOR SELF-DISCOVERY IN HARUKI MURAKAMI'S NORWEGIAN WOOD

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#### ABSTRACT:

This research on Murakami's *Norwegian Wood* focuses on the exploration of memories as a trigger for self-discovery to portray the relationship between memory, identity and personal growth. Through the examination of characters' recollections and reflections on the past experiences, it can be hypothesized that memories serve as a catalyst driving the characters to seek self-discovery. The research indicates that memories are a source of both emotional pain and strength that becomes a medium to confront loss, trauma, grief and in finding resilience in the face of adversity.

**Keywords:** Murakami, *Norwegian Wood*, Memory, Trauma, Identity, Self-discovery.

Memory Studies emerged as an interdisciplinary field in the mid-20th century originating from various academic disciplines concerning psychology, sociology and philosophy. The sociological perspective of

Maurice Halbwachs became a fundamental phenomenon in memory studies in literature. Literary critics analysed how literary works constructed a narrative framework to represent memories of traumatic events and their lasting impact on human psyche. The concept of “Memory” in literary writings, is a dominant theme to explore the complexities of human experience and to analyze the impact of the past in the present to determine individual and cultural identity. Authors examine the influence of memory in defining identity and relationships, also revealing the link between personal and shared recollections. A literary work constructed by considering the psychological, cultural and social dimensions of the memories sustained offers deeper insights in determining identity, exploring the lasting impact of traumatic events and unfolds the quest for self-discovery.

One such notable work is Murakami’s *Norwegian Wood* (1987) focusing on the complexities and impact of memory. This novel is taken for research to analyze the concept of memory and the journey of retrospection that follows any traumatic bereavement. As memories re-emerge revealing the traumatic experiences, human relationships are either shattered or restored. When new possible triggers for memories are encountered, life becomes nothing more than a piece of nostalgic music. Past memories redirects the characters’ present into the path of self-discovery. The quest of meaning undertaken reveals the purpose of life, and the findings of each character differtill the end.

*Norwegian Wood* (1987) recounts the life of the thirty-seven year old Toru Watanabe reminiscing his adulthood. Toru firmly portrays himself as a nonchalant, who really doesn’t care about where he lives or what he studies. Toru’s dull life gains a ray of light when he coincidentally meets Naoko, his love interest and the girlfriend of his late- best friend Kizuki. Toru after his friend’s death,

grieves the sudden loss of human connection and detaches himself from developing any new relationship. Naoko in her present struggles to deal with the loneliness caused by the death of her lover from her past. Toru defines this relationship, between him and Naoko, as a medium “to heal our wounded spirits” (Murakami 33) and finds a striking comfort by spending time together and they grow closer by not mentioning the memories of Kizuki.

These echoes of past influences Toru’s interactions with other characters revealing the force of his memories. Naoko becomes the trigger of these memories, still his every interaction with her seems to be a sunshine in his dull life. The shared moments of joy, the challenges they faced and the unspoken emotional struggles becomes Toru’s feelings of loss and nostalgic. When Naoko retreats to the solitary sanatorium to heal the emotional wounds of her past, Toru feels extremely empty and becomes a “lost soul” without Naoko.

Toru’s acquaintance with Midori Kobayashi, a fellow classmate is a fateful encounter full of vibrant and unconventional experiences in his dull life. Initially known for her bold personality, the narrative progress reveals her own share of emotional struggles contrasting her honest and optimistic nature. Midori’s bond with her parents was a relationship full of longing in the pain of separation. In her memories, the time she spent with her parents is full of tenderness and warmth in contrast to her lonely present life. By portraying her parents as individuals with their own dreams and struggles, Midori confronts her own challenges with unique vulnerability and determination that fascinates Toru.

The loss of her parents defines certain aspects of her character and influences her romantic relationship with Toru as she expects to find “perfect selfishness” in the search for

love. Toru, in turn, though attracted towards Midori decides to remain as friends as he was unable to forget his haunting past. Still, his presence in Midori's life provide her the support and connection to fill the void created by the loss of her parents. The complexities inherent in their relationship mirrors the realistic struggles evident in any human relationship based on love and human connection.

Toru, in the search for a "real" love, travels deep into the mountains to reach the sanatorium and reunite with Naoko. Naoko's lifestyle in the undisturbed landscape of the sanatorium reflects her lingering thoughts between the haunting memories of past and the mysterious beauty of the nature. Her daily life in the woods echoes her determination to retain the lost connection with the present world. She is no longer a shadow brooding over the past but a living soul adapting the flow of time.

Naoko's retreat to the sanatorium reflects the capacity of human soul to find the lost connection in the most unexpected places. Even more, Naoko is able to develop new relationships with the inmates of the sanatorium challenging her solitary retreat and her initial companionship with Reiko Ishida gradually develops a meaningful connection by understanding the unspoken suffering of their respective souls. Naoko faces her emotional wounds and tries to reconcile the present with her traumatic past memories in the middle of woods. Nature becomes a healer and a possible trigger for Naoko's memories.

Reiko Ishida is a warm and supportive inmate of the sanatorium, guiding people around her in handling their own emotional struggles. Her life took an unexpected turn with the accusations of inappropriate relationship with a student, which led to her professional dreams being crushed. This humiliation led to the sudden end of her music career and the societal judgments resonated

deeply in her daily life. Her past memories are occupied with the recollections of societal judgment and the challenges she faced due to mental health. She retreats to the sanatorium forcing herself into a period of seclusion and self-reflection. Reiko's role as a mentor in the sanatorium highlights the importance of shared experiences to understand human psyche. She becomes an alleviating company for Naoko struggling with the traumatic loss of her loved one. Likewise, Naoko helps Reiko overcome her emotional wounds of the past. Naoko's ability to emphasize with Reiko offer her an opportunity to express herself without the fear of judgment.

The journey of self - discovery undertaken by the characters of the novel, whether physical or mental is the medium to confront the uncertainties adjoining memories shaping an individual's identity. For instance, Toru's memories of loss is a pivotal instance that made him led a solitary life and in turn which prompted self-discovery. In his journey of retrospection, the subjection to diverse ideas, people and experiences at university, broadens his outlooks about life and love. Even his purpose to forget the grief ends useless, as memories remain deep inside as a "vague knot of air". Realization hits when he was able to express "knot of air" in the form of words: "Death exists, not as the opposite but as a part of life" (Murakami 30). Until then, Toru considered death as something entirely different from life but death was not opposite of life, it co-exists with life and all efforts to forget about the dead and the grief that follows can only end in vain. This discovery led him to shape his "real" self by confessing his love for Midori and accepting the need to have relationships to find purpose in life.

Naoko's self- discovery arises from her retreat to the countryside sanatorium to find solace and her "real" self in nature, away from the haunting past. Her unresolved

traumatic experiences and repressed memories trigger intense emotional responses and destabilize her mental state. When she enters the sanatorium to cope with her mental struggles, the therapeutic environment and her interactions with other residents gives her the confidence to discover herself. Kizuki's death incorporated "the power of dead" in her but the subtle balance of isolation and connection she receives from a world "inside" the sanatorium, helps to navigate her journey in the exploration of love, loss and personal growth. As a result, Naoko no longer sees herself as "weird and twisted" but starts to believe in the power of connectivity and love. To Naoko, nature becomes the intermediate haven for self-discovery and retaining her lost identity. Her journey progresses but ultimately ends with death.

Reiko symbolizes the resilience and self-awareness gained by an individual in the journey of self-discovery. Her career as musician, full of hardships stimulates her to isolate from the "outside" world and retreat into the sanatorium seeking solace within nature. It is during this isolation that she realizes another alternative path to redemption with the healing power of music. Music, once her source of substantiation changes into a personal asylum for Reiko. Reiko's oneness with music reflects her progress in emotional well-being and signifies the transformative power of art in the process of self-discovery. One important aspect of her journey is the reconciliation with her past traumatic memories highlighting the significance of accepting and confronting such memories to make one's retrospection even more deep and profound. Reiko's interaction with the residents of the sanatorium through her music, their diverse personalities and the shared pain becomes a catalyst for self-exploration, forcing her beyond the confines of past traumas.

Midori is a significant character in the

novel to have a fixed purpose to discover in her journey of self-discovery. Her journey takes a poignant turn as her relationship with Toru becomes a focal point of self-discovery. Being with Toru, contrasting to her vibrant personality, her resilience disappears as vulnerability and loneliness surface. Toru's own complexities in a love relationship induces Midori to reveal her fear of abandonment and the difficulties in opening up emotionally. Her candid conversations about her experiences with Toru provide a clear vision into understanding her evolving "self".

Through the various challenges she overcomes while dealing with Toru's complex emotionality, Midori begins to embrace her individuality. Her ability to empathize and support Toru endlessly, gives her the "selflessness" love she expects from other.

In this novel, Toru embarks on a journey characterised by loss, relationships, connection with nature and personal growth from adolescence to adulthood and in the end, emerging as a more self-aware and resilient individual. Nature's inherent tranquillity and beauty becomes Naoko sanctuary for healing her wounded soul. Being surrounded by nature provides Naoko the necessary isolation from societal expectations to explore her thoughts and emotions more freely. Reiko's musical encounters with other characters further amplify the role of music in the path of self-discovery. The collaboration of nature and music provides her, shared experiences, fostering a sense of community and support needed for self-exploration. Midori's selfless love discovered in the process of retrospection is characterized by endurance and patience as she maintains her independence while expressing her true love to Toru.

The journey of self-discovery is a continual process of growth, learning and an ongoing exploration of the self, to get a clear understanding of what constitutes a purposeful

and meaningful existence. However, some characters like Naoko are unable to escape the underlying sense of meaninglessness in life. To characters pursuing the quest for meaning, memories are a pivotal catalyst triggering the powerful journey evoking emotions, beliefs and past experiences defining identity. An individual confronting his/her painful memories are presented with an opportunity for healing and transformation to reclaim dominance over their lives. Memories uncover forbidden truths and confront unresolved emotions to initiate a profound journey of exploring one's real self.

Future research in this novel can be done on the plot set during a significant social change in Japan could be expended to analyze the cultural reference, societal norms and historical events mentioned in the novel, examining how these factors shape the characters' experiences and identities. Murakami's fame for the use of literary allusions

and references in his novels can be used to trace the intertextuality of other literary works on the novel. The research could be on analyzing specific references to Western literature, music, and philosophy, and examining how they enrich the novel's themes and narrative.

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### **UNVEILING THE CROOKED MIND: EXPLORATION OF THE DARK TRIAD THEORY IN AGATHA CHRISTIE'S CROOKED HOUSE**

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#### **ABSTRACT:**

Agatha Christie's *Crooked House* illustrates dynamism of the criminal and the victim. The Dark Triad Theory propagates the proximate cause of the crimes committed by the criminal via the indirect abetment of the puppeteer. This paper focuses to uncover the supposed victim as the real criminal and the oppressed criminal by the society as the pathetic victim. Aristide Leonides and Josephine Leonides are the still water that runs deep who hold mysteries that the family members are unaware of. The pivotal point of the research paper is to look into the triggering factors that made the criminal to commit crimes and also the dark shadow that never leaves the side of the pathetic criminal who is later revealed to be the chieftain of the crimes. The convergence of the novel, *Crooked House* and the Dark Triad Theory provides a voice for the condemned criminal.

**Keywords:** Dark Triad Theory, murder, criminal, victim, and instigator.

The Agatha Christie classic thriller *Crooked House* (1949) centres on a tragic family mystery. The Leonides are a sizable, dysfunctional family who reside in a sprawling home. That is, until Aristide, the family's head, is fatally injected with a barbiturate. Naturally, suspicion is directed



against the elderly man's young widow, who is 50 years his junior. But Charles Hayward, the engaged to the late millionaire's granddaughter, tries to unveil the mystery in the story.

Machiavellianism, Narcissism, and Psychopathy, collectively known as the Dark Triad, are three unfavourable personality qualities that share emotional aloofness, deceitfulness, and aggression. Canadian psychologists Delroy Paulhus and Kevin Williams, first proposed the term in 2002.

“The Dark Triad consists of three overlapping, yet distinctive personality traits; Machiavellianism, Narcissism and Psychopathy.... All of the three traits are characterised by a disregard of social norms, which often leads to social transgressions in terms of lying, cheating, manipulating and stealing.... The Dark Triad can have toxic consequences for others who are in the radar of those high in these traits” (Lyons 2).

The victim and the criminal's mentality are segregated here due to the colossal similarities between them. An additional character has also been researched for fanning the flames of isolation and anger in the victim. The paper seeks to identify the triggering elements that provokes the victim and explores the personality aspects of both the victim and the malefactor. Awareness and ignorance of both the stimulators cause reckless behaviour in the victim. Dark Triad theory also enables to explore those novels with open endings which lead to the exploration of the actual criminal and the pathetic victim.

Josephine Leonides in Agatha Christie's book *Crooked House* inherits all of the family's negative characteristics. However, the family finds it difficult to accept that one of its own members is so morally depraved as to murder the elderly patriarch Aristide Leonides. Brenda, Leonides' wife, and Laurence Brown, Brenda's lover, are detained for the murder. There is some

scepticism that the actual murderer had been apprehended. No one seems to think Brenda and Laurence are the true killers, despite the fact that the evidence points in their favour. Because if Brenda and Laurence are innocent, then one of the family members must have committed the crime and hence the family is paralysed by its own uncertainties.

All family members are aware that Aristide had been poisoned by his eye drops, which was fatal if consumed internally. Someone had filled an insulin bottle with the eye drops after emptying it. Due to the lack of strong evidence against any family members who knew about the eye drops, the murder inquiry does progress well. Brenda, Aristide's young wife, admits to giving him his shot just before he passed away, but she says she is innocent. Others in the family do not seem to have any justification for wanting him dead.

The main issue in the novel is that Josephine is identified as the villain when she struggles to fit in the family as an intellectual. Aristide, the propagator of all the crimes in the novel, is to look as a good father who cares for his children, a righteous man who lets his daughters-in-law and his sister-in-law become accomplished women in the society and a lovely grandfather to his grandchildren. He fails to be a good grandfather to Josephine and Eustace. Sophia has always seen the world through rose coloured glasses whereas for Josephine, the real world is a poignant sphere.

Red herring of the novel is Josephine as she is calculated as the murderer in the end when Aristide is the real cold-blooded murderer who plays with humans and their emotions as his puppets. The Dark Triad Theory acts as a catalyst to evaluate the deeds of the characters and brings justice to the tarnished one, Josephine. This theory does not justify the crimes of Josephine but it gives a stand for Josephine to explain her reasons behind the act. The Dark Triad Theory also brings to light the danger that lurks behind the

dark, Aristide by analysing every possible scenario.

Aristide Leonides' death is the starting scenario for the novel. His death is assumed to be a murder. Looking back into his life history, it is noted that he is a Greek by birth, but he came to England to escape and to survive. He lived in Smyrna. No reason is given for his conduct of stabbing two men. Later, he comes to England in 1884, aged twenty-four and start his business. He opens a restaurant in Soho. Later, he enters into catering. His business flourishes and he fell in love with an English lady who later became his wife. They had eight children, but Roger and Philip are the only surviving ones in the story. Sophia again quotes, "and then there was my grandmother. I only just remember her, but I've heard a good deal about her. I think she might have had the ruthlessness that comes from having no imagination whatever" (Christie 42). It is also notable that Aristide's wife loved fox-hunting. The couple's ruthlessness has continuously flown into the family throughout the ages. The first generation seems to be self-absolved. But the second and further generations seems extreme in everything they do. Aristide is a calculative man. His actions towards his children and grandchildren can be considered as partiality.

The Five Factor models of Machiavellianism, underlines three of the characteristics that an affected person would certainly have. They are Antagonism, Playfulness and Agency. Aristide and Josephine also have less ideological commitment because they never fear God, or societal rules. Aristide Leonides is introduced by Chief Inspector Taverner as a person who "never made any mistakes in anything he handled" (Christie 25). Being the head of the Leonides family, Aristide strives to keep everything and everyone under his control. He has a low level of agreeableness and conscientiousness. No matter how many people insists on sending Josephine to school,

Aristide is very much rooted to his decision of not sending her. "I've said so to Miss Edith and she agrees- but master would have it that she was best here in her home" (Christie 160).

The reason could be that his secret that Josephine is aware of, could remain contained if she remained home. As he is not so honourable, because he had already stabbed two men out of rage, he cannot stand any insult and he would do anything to make his reputation stand in the society. Sophia says that Aristide had casually mentioned about the ill deeds he had committed in Smyrna. Letting about his deeds casually cannot be taken leisurely as it could be considered as a trap to make Josephine fall in it. Knowing the curious attitude of Josephine, she would have definitely searched for other crimes he had committed and would have found it out. In order to escape the shame, Aristide home-schooled her to contain his dark secret.

"Once when he was telling us about his boyhood in Smyrna, he mentioned, quite casually, that he had stabbed two men. It was some kind of a brawl- there had been some unforgivable insult- I don't know- but it was just a thing that had happened quite naturally. He'd really practically forgotten about it." (Christie 41-42)

Giving the blueprint of his death cannot also be treated casually. He would make himself a bait to lure Josephine and to catch her during her action. But unfortunately, Josephine does succeed in her plans. "Crooked, yes- but not a crook. Never anything outside the law" (Christie 25). He does commit crimes even after settling in England but they are schemed impeccably. He does everything round the law by finding loopholes. Edith states that, "This was the Original Crooked Little Man who had built the Crooked Little House" (Christie 86). The crooked master had allies from Greece even after his departure from there. His will is actually sent to Mr. Agrodopolous who was once helped and befriended by Aristide. This

indicates his calculative behaviour and also his intelligence in settling scores among and out of his family.

“He was about the best judge of anybody in the house” (Christie 186). Aristide acts as the catalyst of the family just in order to manipulate them. Had they had their own freewill to do anything they want, his dominance would have faded away. So, he behaves as a judge among his family members, ruled over their decisions and made them follow his command as a divine saying. He does not allow Philip to fund Magda’s play as he knew that play would be a flop. He appears to be a good father by letting Roger take care of Association Catering. Knowing that the business was going down, Aristide acts to help Roger to build back the business, but before he does so, he dies unfortunately.

Josephine Leonides is the youngest member of the Leonides and murderer of her hateful grandfather. Analysing her in accordance with the Dark Triad Theory, proves that she falls under all the core features: Machiavellianism states that a person with a personality disorder tends to lack in interpersonal relationships as they are the prime manipulators and they never empathize with their victims. They do not have concern for conventional morality because they have the ability to lie and cheat. As they have fixed view of the world, they ignore psychopathology completely. They are prejudiced and they never think of themselves at fault. Same goes with Josephine. She manipulates Charles Hayward when he comes to Three Gables. She tries to act intelligent and yet discloses to him that she knows who the murderer is. She begins to give hints to Charles Hayward as she wants Charles Hayward to reach the conclusion that she wants to derive. She tells to Charles Hayward that she wants to be a detective and he gives in to all her questions about detective works. “We like detective stories. I’ve always wanted to be a detective. I’m being one now. I’m

collecting clues” (Christie 108). It is also a trick played by Josephine to know the next steps of the police. She is portrayed as extremely cautious but her act brings to light her true self.

Antagonism in Josephine is picturized when her dear diary is found and read. All her ill deeds are noted with exact dates. Her aspirations of wiping out the entire family is the ground-breaking news. “Today, I killed grandfather” (Christie 293). This statement is all that is needed to prove her vicious actions against her family. It is stated usually that the first crimes of the juveniles are petty thefts but here, Josephine has killed her grandfather. The extremity is high which denotes how mentally tortured she is in the story. The reason she kills her grandfather is not so levelled. It seems to be lame. “Grandfather wouldn’t let me do ballet dancing so I made up my mind I would kill him. Then we should go to London and live and mother shouldn’t mind me doing ballet dancing” (Christie 299). Josephine is in love with doing ballet dance, but Aristide does not allow her to do so because he is aware of Josephine’s character and her evilness. That is the reason she is home-schooled. “She ought to have gone to school and had children of her own age to play with. I’ve said so to Miss Edith and she agrees - but master would have it that she was best in her home” (Christie 160).

Josephine’s deliberation and orderliness draws her playfulness. She is an ardent listener, a curious child with a diabolic personality. Her motive of killing her grandfather is anchored when he himself gives the blueprint for his death. When Brenda, Aristide’s second wife is giving him Eserine, Josephine questions the reason why the eyedrops should not be taken in a bottle. Aristide’s reply is very helpful for her to execute her plan. He says that if Eserine is injected in the place of insulin, he would turn blue and die because he has a weak heart. Knowing the method of killing her

grandfather, Josephine sneaks into her grandfather's bathroom. She wears gloves, not to leave fingerprints and exchanges the solutions cautiously. Her mode of planning and executing are impeccable.

What pushes Josephine to commit such a crime is the restriction imposed on her to move ahead with her life and passion. She is even good enough to manipulate the course of events. She is just like her grandfather, "Sometimes I think that child isn't right in her head. She has horrible sneaky ways and she looks queer..." (Christie 99). Aristide also has his own ways of doing things. She is also an attention seeker. She just goes everywhere around the house and tends to act like collecting evidences but actually she is leaving fake evidences. Even her fake evidences are taken into consideration by the police. It is indeed ironical when the novel posits: "Child's evidence is always the best evidence there is" (Christie 148).

Josephine's diary reveals her pathological condition. She considers herself hyper-intelligent and unstoppable. In her diary she mentions:

"I hate Nanny... She's making mother send me abroad... I'm going to kill her too -I think Aunt Edith's medicine would do it. If there is another murder, then the police will come back and it will all be exciting again. Nanny's dead. I am glad. I haven't decided yet where I'll hide the bottle with the little pill things. Perhaps in Aunt Clemency's room - or else Eustace. When I am dead as an old woman, I shall leave this behind me addressed to the Chief of Police and they will see what a really great criminal I was." (Christie 300)

She is instilled with greater confidence that she could escape till the very end without causing dubiety. She must have inherited this trait from her grandfather as once he killed two men. The ruthlessness is passed down to the generations. In genetics, it is said that grandchildren inherit their grandparent's traits. The predominant component Josephine could

have inherited could be the negative ruthlessness. Her ruthlessness is triggered dreadfully when her grandfather stands as a hindrance in fulfilling her wishes. He always restricts her from doing the things she wanted. Aristide only favours Sophia as he knows her talents. Favouritism knocks down their relationship as a grandfather and grandchild.

Another trait of healthy narcissism is rule breaking. When the news of her grandfather's death breaks out, Charles calls her for a meet. The police had instructed the family members not to step out of the house till the investigation gets over, but still Sophia flies away to meet Charles. "But I was determined to see you.... I got out of the bathroom window and shinned down the water-pipe" (Christie 19). Though a policeman trails behind her, she does not care about it. All she wants to do is to meet Charles.

Josephine is not celebrated in the novel. When she claims to know the murderer, Charles Hayward approaches her for clues. She leaves a faux clue that leads to the whole misconception. She always wants to be in the limelight. When Laurence and Brenda get arrested, she feels everyone would go back to their ordinary schedule. In order, to stir problems and be the Sherlock Holmes of Three Gables, she kills Nanny. The goblin-like girl wants to be in the centre. When Charles Hayward realizes the true nature of Josephine, he accepts what Sophia said was true. "What Sophia had called the 'ruthlessness of the family' – had met together" (Christie 297). Aristide had already realized that Josephine was the only source of danger for the family.

The theory of the Dark Triad rules over Josephine. She feeds on people to get compliments. She is portrayed as an ardent murderer who loves to kill people with simple tricks. She does not use sharp weapons and that proves a winning point for her. Blood spilling would always leave for clues and fingerprints. Easy techniques make her not to

be enlisted in the suspect list. Josephine must have experienced Alexithymia, the inability to understand oneself. If, thus exhibiting unempathetic emotional responses. It is not evident whether she is happy after committing two murders in a row, as she is more focused on boasting her knowledge of who the killer is. Her level of happiness is not measured in the novel but she is contented about her killing. Her state is caused by genetics and environment.

Aristide Leonides is the mastermind behind all the occurrences both in family and business. He fits into the theory of the Dark Triad perfectly than the Leonides' sisters as he is the emitter of the tenets of the theory of the Dark Triad among his family members. Sophia admits:

"I think that's what I mean when I said we all lived together in a little crooked house. I didn't mean that it was crooked in the dishonest sense. I think what I meant was that we hadn't been able to grow up independent, standing by ourselves, upright. We're all a bit twisted and twining." (Christie 182)

Aristide does not let anyone grow up independently as he wants to keep them coming to him for everything. He makes them grow all twisted and twined. His crooked mind is always focused on his self-growth than his family's. He loves to be the centre of attention just like Josephine who goes around with her diary writes down everything and acts like she knows everything. Eustace states: "... writing things down in a little black book and pretending that she's finding out a lot." (Christie 195). The way she allures Charles to a conclusion is all inherited from her grandfather. His venomous character divulges in reality in Josephine's behaviour. He is her triggering point. Her unsteady and malicious self is because of her grandfather's attitude and his treatment towards her. Had he thought about his family and his grandchildren's life, Josephine would have turned a new leaf than being a cold-blooded

murderer. She ruins herself by giving into her agony and revenge. People around her fails to notice her because Aristide makes Josephine and Eustace his wards.

According to statistics, 150,000 children in England's biggest cities are either currently criminals or are in the process of becoming one. These kids, who are either orphans or children who have been completely abandoned by their parents, encounter other criminals who are farther along in their careers in prison or in the common hangouts of these homeless wanderers, and are encouraged by them to commit additional crimes. The sociological perspective is exactly that of the mid-nineteenth-century reformers who steadfastly brought to the public's attention the impacts of subpar housing and a lack of sufficient educational opportunities and recreational facilities. The homes of aristocrats contained more specific horrors related to the issue of adolescent misbehaviour. The Dark Triad Theory the grand unfolding of the puppeteer and the marionette in the story.

Agatha Christie's *Crooked House* makes the Dark Triad Theory to have a fortune in explaining its area of expertise with Machiavellianism, Narcissism and Psychopathy to let the world know that a story of a criminal has an un-identified ring leader and an ignorant stimulator who directs the victim alias the criminal for the world, to carry out the crime.

As it is said in the *Holy Bible*, "Whoever causes one of these little ones who believe in me to sin, it would be better for him if a great millstone were hung around his neck and he were thrown into the sea" (Mark 9:42). The one who leads others to commit sin is the sinne.

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## **RACISM REFLECTED IN TONI MORRISON'S THE BLUEST EYE**

**Ms S.Sushmitha**

### **ABSTRACT**

The aim of the research is to bring to light the subtle ways of racism followed by the individuals and the overall racism, the society practices through Toni Morrison's novel *The Bluest Eye* by using Critical Race Theory as the theoretical framework. The Introduction presents a brief history of African American Literature, the biography of the author of the novel *The Bluest Eye* and the Principles of the Critical Race Theory. The study further discusses the indicators of racial segregation and disparity in the novel *The Bluest Eye* brought out by the principles of the Critical Race Theory such as Everyday Racism, Material determinism, Social Construction, Voice of the color and Intersectionality or Racialization. The study also showcases how the voices of the minorities are often neglected and it stresses the use of one's voice which can be considered as the greatest weapon by its fourth principle Unique Voice of the Colour and the study also focuses on the effect of racism which makes an individual to change their life. The research concludes by summing up the discrimination faced by the black people in terms of economic disparity, social prejudices and injustice and it also

emphasizes on the use of minorities voices as a crucial tool in achieving change for the marginalized groups of people.

**KEYWORDS:** Racism, Prejudice, Voice

### **INTRODUCTION**

Literature, originating from the Latin word *litteratura* meaning writing formed with letters, is a gateway to learning patterns and expressing emotions and ideas. It is a method of recording, conserving, conveying the knowledge, entertainment, and involves roles of social, psychological, spiritual, or political. Literature can be classified according to language, origin, period, genre, and material. African American Literature focuses on the lives and folklore of African Americans, beginning during the 18th and 19th centuries with writers like Phillis Wheatley, Frederick Douglass, Langston Hughes, Zora Neale Hurston, James Baldwin, Ralph Ellison, Gwendolyn Brooks, Lorraine Hansberry, and others. This literature emerged in the US and focused on American culture, racism, slavery, equality, and society.

In the 1970s, black writers became part of African American Literature, with books receiving best-selling and award-winning status. Toni Morrison, an American novelist, won the Nobel Prize and Pulitzer Prize for her works, including *The Bluest Eye* (1970), *Sula* (1973), and *Beloved* (1988). She was the first African American woman to receive the Nobel Prize for literature in 1993 and established the Princeton Atelier in 1994 to help students create their own work. One of Morrison's most notable works is *The Bluest Eye*, set in Lorain, Ohio, which depicts the struggles of a young girl named Pecola Breedlove who believes whiteness is beautiful and having a blue eye is beautiful. The play explores the societal expectations that if the marigolds live, so will Pecola's baby. In conclusion, literature is a universal language

that reflects the traditions, cultures, and problems of different countries. It can be classified into various forms, such as non-fiction genres, and is influenced by various factors such as language, origin, period, genre, and material.

The *Bluest Eye* is a novel that explores racial inequality and discrimination, highlighting the complexities faced by characters, even small children. Critical Race Theory (CRT) is used to explore the racial colors and the embeddedness of racism in legal systems and policies. CRT emerged in the mid-1970s and spread to other fields like gender studies, ethnic studies, education, and sociology. Toni Morrison's novel, *The Bluest Eye*, addresses racism through the difficulties faced by characters, such as Pecola, who is not treated properly by her family due to white discrimination. The novel was written during active Civil Rights movements, and Morrison felt that her voice was not being heard. She incorporates issues of gender into her work, incorporating racial sensibility and gender issues into her narrative.

This research explores racial elements in *The Bluest Eye*, highlighting how racial discrimination leads to factual mistakes and complexities among characters. It emphasizes the importance of self-esteem and equal treatment, emphasizing the tragic flaws in the play.

## LITERATURE REVIEW

In an article „*Racism: Toni Morrison's The Bluest Eye a Mouthpiece of Cloured People*“ by Nisar Dar and Mohd Mir the writers highlight on the struggles of the coloured people and experience in a predominantly multicultural post-colonial white Americas.

In another article by Issam El Masmodi *The Concept of Race in Toni Morrison's "The Bluest Eye"* several issues that concern the

black race in the light of the novel *The Bluest Eye* is discussed.

*The Impacts of Racism on Pecola Breedlove's ways of seeing herself in Toni Morrison's The Bluest Eye*“ by Sarjana Pendidikan in this article it foregrounds on two problems (1) Racism portrayed in Toni Morrison *The Bluest Eye* and Impact on racism in Pecola Breedlove character.

In *Racism within African American Communities in Toni Morrison's The Bluest Eye*“ by Maria Bring, the writer has focused on how racism, marginalization, and oppression have oxidized into the characters self-esteem and the sense of self-worth.

## ELEMENTS OF RACISM IN TONI MORRISON'S *THE BLUEST EYE*

*The Bluest Eye* is a novel that highlights the impact of racism in the mid-20th century, particularly on African-American people who have been unable to break free from their social and psychological inferiority complex. The novel portrays the lifestyle of black people who try to personalize the culture of white people but feel rejected in every way of life. Toni Morrison supports African-Americans in finding happiness in their own culture and color, but racism remains a dominant figure in society.

Richard Delgado and Jean Stefancic's Critical Race Theory is used to analyze the text and unravel the shroud of racism. They propose five principles for this theory: Everyday Racism, Material Determinism, Social Construction, Voice of the color, and Intersectionality or Racialization. These principles are applied to analyze the text and find the underlying and innate forms of discrimination in *The Bluest Eye*.

Everyday Racism plays a vital role in the novel, as it shows how society has been harsh on African-American people, especially through the characters Pauline Breedlove and Pecola Breedlove. The novel portrays the emotional battering and exhaustion

experienced by the black characters, highlighting that people are not born with the intention to hurt others but are taught to do so when they themselves are hurt.

Pecola Breedlove, a submissive and mysterious character, is given the blue eye to change her desires and face the pain of her existence. However, her fate is worse than death, as she moves to the edge of the town where she can still see her. *Everyday Racism* plays an essential role in the novel, as seen in the characters' interactions with Mrs. MacTeers, Claudia, and Maureen. Claudia encourages Pecola to be bold and face the world, while Pecola struggles to overcome her inferiority complex.

In the novel "Everyday Racism," a mother and her son, Lousis Junior, live in Lorain, where they are taught the distinction between the Cloured People (White People) and the Niggers (Black People). Lousis Junior wants to play with black boys and bully the girls pacing the playground, but he does not want to choose the niggers girls. He sees Pecola walking alone and invites her to play with him. However, Pecola is afraid of Junior and refuses to play with him.

Cholly Breedlove, the father of Pecola, rapes his own daughter due to white men tricking him into performing sexual deeds in front of them. This leads to Cholly experiencing genuine suffering and humiliation at the hands of white men. As a result, he falls apart when his freedom becomes complete.

The MacTeer family faces various issues within society, including the domination of Mr. Henry, a hardworking and quiet man who wants to gain trust among the children of MacTeers. The novel illustrates the cycle of oppression, with characters born in purity and innocence being degraded by the dominance of society.

The second principle of Critical Race Theory, Material Determination, is used to illustrate the domination done at the work

place between whites and blacks. The play "Zicks Coal Company" highlights the material determination of people in society and their own homes. Claudia and Frieda face numerous issues at work, and when they fall, they are asked to pull off their pants and go home.

The author Toni Morrison compares the Marigolds with the black people, highlighting the unyielding nature of the earth. The novel uses ancient folk wisdom to convey the correlation between human events and natural events, emphasizing the importance of addressing everyday racism and the struggles faced by individuals like Pecola and Cholly Breedlove.

In the novel, Claudia and her sister Frieda experience poverty and racism, as they see their neighbor Rosemary Villanucci, who only eats bread and butter for white people. They have mixed feelings of hate towards Rosemary and are given a brown stocking and cod-liver oil to boost their immune system. The color of their dress symbolizes their belonging to the black community.

Mr. Henry, a paying guest of MacTeer, marries Miss Della Jones, causing false assumptions about his intentions. The people of the place believe that Henry will marry Della, but this is false. The Breedlove family lives in Lorain, Ohio, and is isolated and separated from the community. Pauline, a black woman, works at the Fisher Mansion, escaping her hardships and adopting the oppressors' discourse, particularly the discourse of physical beauty.

A West Indian preacher named Soaphead Church, raised by wealthy academics, councils people who come to the church. Pecola Breedlove asks for Blue Eye, a petition he believes God has designed an imperfect universe. Soaphead insults Pecola and gives her poisoned food, threatening to give her blue eyes if she does the task. Pecola does the task, and the dogdies.

The effect of racism on the characters in



the play is evident in Soaphead's letter to God, asking for pardon for his misdeeds and mentioning the beast of girls.

Pecola, the protagonist, does not attend school due to societal rejection and her experiences in the classroom. Her family is separated, and her father Cholly Breedlove is considered inferior by the community. The novel highlights the material determination of racism, with white people dominating the black community and causing no one to dream. The African American community struggles to consolidate their weaknesses and maintain their property, while whites have more access to it.

The third principle of Critical Race Theory, Social Construction, states that race is not biological or natural but is socially established and contrives ideas through false stereotypes. This socially invented idea is often negative and prejudicial, leading to discrimination and violence. Pecola's experiences with white beauty standards, such as the blue-eyed dolls and the belief that light-skinned women are more beautiful, are depicted in the novel.

From the beginning, Pecola has a negative self-image, with mainstream society and her mother denying her beauty. She wishes for a blue eye to look beautiful, as she believes that her own eyes should be blue like Alice. The novel provides an extended depiction of how internalized white beauty standards deform the lives of black girls and women, with whiteness being the only mirror that helps up to the Black community.

In conclusion, Pecola's story highlights the impact of racism on the lives of black people and the importance of addressing the material determination of racism. In the novel, Pecola meets Maureen Peel, a white classmate with long hair and green eyes.

The white community is pleased with Maureen, but not the black children. Boys are cruel towards Maureen, even naming her "Six - Finger - Dog - Tooth - Meringue - Pie." Pecola is ridiculed by boys and takes the class

to an ice cream shop. Maureen then shouts "I am cute!" and "Ugly!" to Pecola, leaving her feeling isolated. Claudia and Frieda advise Pecola to be bold and face the world, questioning why people think Maureen is cute and not cute.

The novel also mentions different towns, Akron, Mobile, Meridian, and Negadoches, where girls are tall, narrow, and hollyhocks. Garaldine teaches her son about racial discrimination, making his son Louis Junior torture Pecola. Pecola's mother, Pauline, is bullied by her family and experiences racial neurosis. She learns to judge people based on appearances from Hollywood movies, leading to her subversion of self-esteem and becoming a victim of dominant cultural standards.

Social construction plays a crucial role in the novel, as physical appearance is given importance in every place they go. Pecola internalizes her desires for blue eyes, leading to her tragic fate of having The Bluest Eye. Pauline's servile reverence to the white ideal of beauty makes her see her daughter "ugly," while Claudia and Frieda do not yield to the image of white beauty. Overall, the novel highlights the importance of social construction in shaping characters and their experiences.

The Voice of Color, a principle in the Critical Race Theory, emphasizes the unique voices of minorities. In *The Bluest Eye*, the protagonist, Pecola, struggles to speak up and express her feelings due to racism. Her silence and dread make it difficult for her to express her anger. Claudia and Frieda, who interpret Pecola's actions, try to change her mood and make her feel less loved. When Maureen insults Pecola, Claudia and Frieda yell at her, causing her to feel irritated. Claudia's voice reveals the flaws of the community members who elevate Maureen Peals, ignoring their meanness and sense of superiority. The Critical Race Theory emphasizes the importance of recognizing the voices of minorities and their struggles to overcome

discrimination and oppression.

In the novel, Pecola is portrayed as an ugly person by her classmates and teachers, leading to her anger and self-consciousness. She struggles to express her feelings of ugliness and the domination she faces in society. The novel also highlights the importance of Intersectionality, which focuses on people's change in racist societies.

Pecola's mother, Pauline, undergoes significant changes after seeing a Hollywood movie and focusing on her physical appearance. She becomes more concerned with her physical appearance and dreams of her future, while her mother, Mrs. MacTeers, becomes more interested in her physical health and the benefits of drinking whiskey.

The protagonist, Pecola, eventually achieves the desire for blue eyes, despite being told she is ugly from birth. She prays every night to have them, and her interactions with prostitutes and Maureen further complicate her feelings. Junior Louis tortures Pecola, leaving a black cat on her face, and she seeks the bluest eye at Soaphead Church.

The novel highlights the ongoing racial segregation and disparity in society, highlighting the importance of minorities and the domination done towards black people. The five principles of Critical Race Theory, Everyday Racism, Material Determinism, Social Construction, Voice of Colour, and Intersectionality, are used to highlight the societal issues and the need for change.

## CONCLUSION

The Bluest Eye is a novel that focuses on racism and its impact on individuals and society in the mid-20th century. The novel explores the experiences of Pecola Breedlove, who seeks the bluest eye to be beautiful, and Cholly Breedlove, who commits a heinous act due to his experiences. The novel uses Critical Race Theory as a theoretical framework, highlighting the origins, development, and pioneers of African American literature.

The novel highlights the five basic indicators of racism: Everyday Racism, Material Determinism, Social Construction, Voice of Colour, and Intersectionality. Everyday Racism highlights the issues of color and their daily struggles, while Material Determinism focuses on the problems faced by individuals in the workplace and educational institutions. Social Construction highlights the negative terms and comments that society associates with black people, such as violence and rape.

The Voice of Colour highlights the torment and torture experienced by Pecola, who is never able to fight back. Instead, Claudia and Frieda raise their voices when Pecola is tormented by schoolboys and Maureen insults, urging her to be bold and brave.

In conclusion, The Bluest Eye highlights the enduring impact of racism on individuals and society, highlighting the importance of understanding and addressing these issues in today's technological age.

The Bluest Eye by Toni Morrison explores intersectionality through the characters' experiences and the societal expectations of beauty. Pecola, Pauline, and Claudia all have an instinct that white is beauty, which plays a significant role in the novel. Morrison uses language to free the reader from its restrictive aspects and creates a fluid prose style that resonates with the entire community.

The novel highlights the deep rootedness of the Afro-American community and the importance of race and gender within it. The author uses multiple narratives, each speaking in an innate tone, to give an insider's view of the Lorain, Ohio community. Pecola's alienation and obsession for blue eyes are evidence of the false and unreal standard of beauty in the black community.

The narrative technique in The Bluest Eye involves various narrative structures, with Claudia MacTeer as the most prominent narrator. She records her own perception and comment, and she introduces temporary

narrators to provide an objective perspective on the events. The omniscient narrator describes Pauline's early life, Cholly's trouble, and Pecola's parents' fights.

The author's vision on issues like racialized, genderized, and sexualized others within the same rooted black community is deeply felt by the author. Through the depiction, the author creates an effect on the counter discourse and dishonest the dominant discourse.

The *Bluest Eye* explores themes of community, whiteness, sexual initiation, abuse, season verses, and ecofeminism. It highlights the importance of social empowerment and the voices of marginalized groups. The novel highlights the clashes within the black community due to white domination, emphasizing the need for self-confidence and addressing individual issues. Further research is needed to explore these themes further.

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#### RETRAUMATIZATION: A PSYCHOLOGICAL APPROACH TO ALEX MICHAELIDES'S THE SILENT PATIENT

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#### ABSTRACT:

*The Silent Patient* is Alex Michaelides's first debut novel that sets out to describe the life of Alicia Berenson who becomes a victim of irregular series of traumatization. It is a psychological thriller that deals with the mystery behind the death of Alicia's husband, Gabriel, and Alicia's sudden silence after the tragedy. The novel has many twists that involves recollection of the past. This novel deals with multiple dimensions of psychological factors. Alicia, the novel's

protagonist faces major traumatic and stressful situations mainly caused by external forces. Theo Faber, the psychotherapist as well as the antagonist in the novel has also undergone mental suffering during his childhood. The research paper concentrates on how Alicia and Theo's childhood trauma that happened in the past gets triggered and how they reexperience the trauma at present. Alicia, with Post-Traumatic Stress Disorder, responded to the triggering situations that vary with the intensity of her emotions when they are relived or remembered. Through this novel, Michaelides points out the vital role of traumatic events in one's psyche. This paper focuses on highlighting the effects of retraumatization and becoming a revictim of the mental illness, Post-Traumatic Stress Disorder (PTSD).

**Keywords:** Alex Michaelides, Unraveling mystery, Post-traumatic stress disorder, Retraumatization, Revictimization, Silence.

Literature acts as a medium that provide insights and understanding to people, thus becomes the mirror of the society. Literature depicts the real world, human traits, their thoughts, feelings, emotions and psychological aspects associating with the traits of the society. Literature and Psychology are inseparable. Psychology is an attempt to study the language of human mind and behaviour to a particular situation. It varies from each individual, and depends on the circumstances. Literature and Psychology, both deals with the societal aspects and human reactions.

Alex Michaelides was born in Cyprus in 1977. He has dual citizenship as he was born to a Cypriot father and an English mother. He holds an M.A. degree in English from Trinity College, Cambridge and an M.A. in Screenwriting from the American Film Institute, Los Angeles. He also studied psychotherapy for three years. He worked as a part-time psychotherapist under his sister's

concern for two years. He can be rightly called a novelist, psychologist, and screenwriter.

Alex Michaelides was fond of reading books from his childhood. When he was thirteen years old, he developed an interest towards Agatha Christie's mystery novels. He was highly influenced by the impact that Christie's novels left on him. He also acknowledged in an interview that he had a happy time reading her novels and that ultimately made him as a voracious reader and a passionate writer. He wished to recreate the experience he encountered while reading Christie's novels to readers. He took Agatha Christie's plot and the narrative structure and infused intensive psychological aspects that are overtly complex.

Psychological thrillers are a kind of narrative that is told from the viewpoint of an unreliable storyteller or a mentally stressed character. The human mind has the super power to find the meaning and intention of what is spoken and unspoken. Psychological thrillers paint a mental picture of the human mind that is in distress and its reaction to the situation. It depicts the mental state of the protagonist and antagonist who are intensely obsessed with violence with significance to their internal conflict or trauma.

The first debut novel by Alex Michaelides is *The Silent Patient* published in 2019 that took the New York Times bestseller in hardcover fiction for almost a year and stood irreplaceable. It was a Sunday Times bestseller for ten weeks. His second novel *The Maidens* was published in 2021. It debuted at number two on the New York Times bestsellers list. Other than novels, the author has written screenplays notably *The Devil You Know* (2013) and *The Con Is On* (2018). He was fascinated in interpreting the great tradition of psychological suspense in British Literature. He liked the tradition wherein there is a sudden shift in the point of view and the readers were made to realize that they were looking all the way wrong the whole time,

which the author experiences in Agatha Christie's *Five Little Pigs* (1942) or *Death on the Nile* (1937).

Alex Michaelides in his novels depicted the psychological trauma and mental depression through his characters' lives. He is a part-time psychotherapist and expressed a great interest in learning the science of the mind. He was highly inspired by Agatha Christie's detective fiction and Alfred Hitchcock for his contributions to thrillers. He incorporated his admiration of Greek mythology and the place where he worked as a part-time psychiatrist in real life as the setting for his novel. He had a profound incentive for Greek mythology. The novel *The Silent Patient* begins with an epigraph, "But why does she not speak?" which was taken from the Greek tragedy, Euripedes' *Alcestis*. This play displays the life of Alcestis from Greek mythology. Admentis, the wife of Alcestis sacrificed her life in order to bring her dead husband back to life. When Heracles gets to know, he wrestles death and brings Admentis back to life. She was not able to speak for three days. The storyline of this Greek tragedy influenced Alex Michaelides to write *The Silent Patient*.

Post-traumatic stress disorder is a mental illness that evolves due to tragic situations in one's life. It may occur due to tragic events like sexual attack, marital violence, physical abuse and any other disturbing events of the past. An individual with this type of mental disorder experiences unhealthy thoughts, feelings and dreams in their life. People who suffer from this particular illness attempt to make many suicides. Sometimes, they lose their minds and become insane beings of the society. Retraumatization occurs when a Post-traumatic stress disorder patient recollects the past tragic events in life through a catalyst who may be an individual, an incident, or an environment. These patients are vulnerable to get reminded of their past at any time. The

patients undergo a lot of mental suffering while experiencing the same traumatic event again at present. Retraumatization makes Post-traumatic stress disorder individuals to lose trust on others. It makes them pessimistic and less enthusiastic about life. They tend to experience flashbacks, hallucinations and nightmares. Suicides play an important role in their life.

The concept of Retraumatization is applied to the novel *The Silent Patient*, as it is evident that both the protagonist and antagonist of the novel undergo the same traumatic mental condition. The research focuses on the importance of repressed memories of the past and their tragic influence on the future. The reen countered buried emotions will have considerably bad reactions in the living present. The research proves that childhood trauma that is suppressed and deeply buried will have a counter-reaction that will cause unimaginable reactions in the life of both parties. The occurrence of retraumatization in the life of the characters will be ventured through the study in detail with the help of trauma theory.

Anke Ehlers and David M. Clark proposed a cognitive model for persistent Post Traumatic Stress Disorder (PTSD). It is a disorder caused because of the feeling of excessive fear, dreadfulness and uneasiness. It is the problem caused mainly due to the storage of memory probably the tormented traumatic event that has previously happened. Ehlers and Clark suggested a cognitive model proposing the persistent PTSD that manifest itself when individuals materialize the traumatic events and its sequelae with the present incident create a serious impact or a threat.

Eshler's and Clark's model covers the major domains which includes Memory which is the past repressive memories of the traumatizing event, Appraisals is an individual's reaction to the situation after experiencing a traumatic incident, Behaviours

is a feeling the sense of current threat naturally leads to the use of coping behaviours, which specially includes Avoidance. Appraisal of the traumatic event can produce a sense of current threat. The act of personalizing trauma intends to certain appraisals with reference to Theo Faber who perceives to think that he deserves to be suffered and it can be clearly seen in Alicia's life when she says that she is deserved to die.

Retraumatization is a condition that results from experiencing a similar traumatic incident that has a great impact when it is reoccured in the present. As Sigmund Freud, rightly said, the emotions that are unexpressed in the past are triggered and comes forth explicitly in uglier ways. The felt emotions whether it is mourning, grieving about the unexpected actions, expressing anger, crying, weeping, and wailing out the inner voice when the agony is at its peak are suppressed at the moment of the traumatic experience, they become hidden secrets at the unconscious and comes out more forcefully that one cannot endure in the present.

In the selected novel *The Silent Patient*, Alex Michaelides employs retraumatization on characters in order to reveal the intense impact of childhood tragic events in an individual's life. Human mind is capable of storing a lot of joyful as well as sorrowful memories. Michaelides points out the serious effects of traumatic events on one's life through this novel.

### **RETRAUMATIZATION IN ALICIA'S LIFE:**

Alicia Berenson, the novel's protagonist, was a famous painter who shot her husband Gabriel five times. The trauma that Alicia sustained during her childhood is awakened by powerful triggers of the present which totally dominate her actions. Alicia as a child was exposed to a trauma that became an unpleasant memory. The mental agony that she experienced was deeply stored inside her.

Out of depression, Alicia's mother committed suicide along with her in the car. As her mother wished, she died. Fortunately, Alicia survived. She hoped to see her father after the accident but his response only gave her pain and anguish.

Alicia's father could not accept that his wife had died and he showed his mental distress to her. Her father killed her psychologically when she was an innocent little girl. He said at her face that he wished she had died instead of her mother. This statement even after her father's demise, the tragic memories stayed and pricked her throughout. The mental agony of her father's dislike towards her was buried and trapped deep inside her. She has never recovered from that emotional trauma. In Alicia's case, it is very evident that, though the person who was the reason for the mental trauma dies and disappears, the hurt and the unexpressed emotions never vanish away. Her unvoiced emotions about her father and the trauma he puts on her when she was a kid ended up hurting Gabriel and ruining her life. The rage she felt for her father who was already dead, was visited on her husband. The buried tragic emotions erupted and she shot her husband five times in the head. Alicia's childhood was stressful with bad memories. She loved her husband unconditionally. When she comes to know of his illicit relationship with other women and her husband's unhealthy decision to sacrifice his wife's life even after realizing that he did wrong, makes her reexperience the childhood trauma.

"I wrote down another word: CHILDHOOD. If I was to make sense of Gabriel's murder, I needed to understand not only the events of the night Alicia killed him, but also the events of the distant past. The seeds of what happened in those few minutes when she shot her husband were probably sown years earlier. Murderous rage, homicidal rage, is not born in the present"(41).

Alicia becomes guilty about her act of killing her own husband. Later, she is sent to a psychiatric ward, Grove, for further investigation. The case becomes well known among the people and everyone starts to call her a 'Silent Patient'. Alicia's trauma reoccurrence in the present made her reach the peak of her mental stability. She as a post-traumatic stress disorder patient could not control herself and becomes mute.

### **RETRAUMATIZATION IN THE FABER'S LIFE:**

Theo Faber, the antagonist of the novel, grew up in life feeling edgy, afraid and anxious. He had a comparatively tragic childhood where he was growing up with his angry dominant father which made him experience anxiety disorder. He was mistreated by his father. "You know, one of the hardest things to admit is that we weren't loved when we needed it most. It's a terrible feeling, the pain of not being loved"(96). He longed for love that was constrained for him. His happy days are the moments he spent with his mother. Later, as he experienced continuous depression, he decided to retreat himself 'physically and psychically'(20).

Depressed of his situation, he tried committing suicide to get rid of all his mental torture. He then realized that his only hope for survival was to get away from his home which he considered a 'semi-detached prison'(20). He was unable to forget his torment memories, rather he internalized it and buried them deep inside his unconscious. He, in order to cure his mental illness, decided to meet the therapist, Ruth. He used to pore out all his painful memories which she patiently listens to. Slowly his traumatic instances get faded and he began living a normal life. Theo Faber tried to fascinate Alicia, but he was unsuccessful in his attempt. He gets completely deprived of his childhood trauma when he is in acquaintance with Kathryn (Kathy). Kathy is Theo's wife. Theo saw

Kathy and at once he fell in love with her. He finds her company satisfying. He found that he was happy and content with Kathy.

Actual trauma in the present started when he discovers the infidelity of his wife. He sets forth to investigate the secret illicit relationship of his wife with another guy. He instead of confronting his wife, feared of getting abandoned again, he becomes angry at the man for corrupting his married life. His childhood mental agony gets reawakened at this moment and captivates his mind directing it in a destructive manner. His mental trauma dominates his actions and he reacts according to it.

Theo reminiscences about his harsh childhood as he treats Alicia, six years after she killed her husband and remained mute hitherto. He was unaware that Alicia was the wife of the man who had an illicit relationship with his wife, Kathy. He starts to find out the life history of Alicia. Suddenly, he discovers a diary that narrates Alicia's relationship with her husband Gabriel. Theo becomes aware that Alicia had immense love for her husband. He feels guilty about being one such reason for Alicia stabbing her husband to death as an act of trauma relief.

In *The Silent Patient*, both Alicia Berenson and Theo Faber are the victims of childhood trauma. They relive and reexperience the mental agony that was similar to what they experienced during their childhood. Alicia could not react when her father showed complete disapproval of her living when she was a child. But at present, when her husband was selfish and wanted Alicia to die at his place, she becomes a victim of the past trauma. The main idea of the trauma in Theo's life is the fear of not being loved, abandoned relationship, forsaken love and deprived emotional strength. They did not have a normal peaceful childhood. They have undergone pathetic situations yet they controlled their agony and suppressed

their hard feelings and became happy, only when they met their love of life.

Alicia's father and Theo's father are the two major people who are the reasons for trauma in their life during their childhood.

Alicia forgot her tormented childhood when she married Gabriel Berenson. Every human is like a 'blank slate' as kids and the memories that fill one's mind depends on what one experience and encounter in one's life. The part two of the novel begins with an epigraph, a quote by Sigmund Freud that says, "Unexpressed emotions will never die. They are buried alive, and will come forth later, in uglier ways"(54). This part of the novel explores Alicia's diary entries wherein the major mysteries of the novel are unveiled. Somehow, she managed to suppress and hide her depressed emotions of the past but it is triggered and she became the sufferer of a similar situation in the present.

The childhood trauma, the anger, the hurt which stayed unexpressed was building up. It was not a sudden decision she made to kill her husband; the emotions have been growing. When one day she couldn't hold it anymore, her resentful emotions prepotent her actions and she killed him. Theo was the veiled man who manipulated Alicia and prompted the murder though Alicia was the murderer. Alicia quotes, "That's the truth. I didn't kill Gabriel. He killed me. All I did was pull the trigger"(277).

Theo Faber is also the survivor of the childhood trauma which he experienced in the past. His guilt and anger grew into a murderous rage. His anger not only affect his life but also Alicia's life. He spied on Alicia's house for a few days which the reader gets to know from her diary entries. He finally goes inside her house, ties her up, and reveals all the truth to Alicia about her husband's disloyalty. Theo forces Gabriel to choose between his life or that of his wife. Gabriel selfishly chooses to sacrifice his wife's life to prevent him from dying. His actions pushed

Alicia to the edge of her own sanity. Theo cunningly played his trick of using Alicia to kill his rival. Despite Theo leaving without killing either of them and the reoccurrence of Alicia's childhood trauma drives her into a spiteful rage. Both Alicia and Theo's childhood trauma triggered the present and influence their actions vigorously thus ending up in conflict.

Alex Michaelides was successful in portraying the characters' mental agony and trauma and how their childhood trauma influenced their actions in the present.

Retraumatization occurs even if the person's past trauma has no connection with that of the present. *The Silent Patient* novel can be further researched on the influence of Greek mythology, theory on archetypal criticism and other aspects of psychoanalytical concepts. Thus, it is very evident that the novel's protagonist, Alicia Berenson, and the antagonist, Theo Faber, become victims of retraumatization of the past childhood trauma when those buried emotions are erupted.

Alicia Berenson's childhood, her relationship with Gabriel, and her sad and happy moments are revealed to the readers only through her diary entries. Alicia's diary entries narrate the plot that happened before the murder and Theo Faber's narration reveals the happenings after the murder. The plot twist is unveiled to the readers in Alicia's narration. Alicia's silence is the manifestation of the severe trauma that she suffered. Her negligence to speak was also a significant reaction to her psychological distress. Theo Faber as a forensic psychotherapist with "ALCESTIS and CHILDHOOD" (Michaelides 44) as clue words proceeds to find the mystery behind her silence.

Alex Michaelides's *The Silent Patient* is a spectacular fiction that reveals the writer's inner potential of exposing the characters' inner feelings. The research primarily focuses on highlighting the occurrence and reoccurrence of trauma, one in the characters'



childhood where they resist reacting rather internalized their emotions, and another in the characters' adulthood where the emotions are articulated directly by the triggered traumatic feelings which are unexpressed in the past.

The concept retraumatization is caused as a result of enduring trauma in the life of Alicia Berenson and Theo Faber. This leads to Post-Traumatic Stress Disorder (PTSD) in both the characters. Their emotions are re-experienced when it finds the matching triggers of the present in association with the past. The intrusive thoughts that have been reprocessed in Alicia Berenson and Theo Faber have negative appraisals of the traumatic event.

To conclude, the research reveals the condition of Alicia Berenson and Theo Faber after they experience the collision between the past concealed and the present exposed trauma. Thus, it is very evident that the novel's protagonist, Alicia Berenson, and the antagonist, Theo Faber, becomes a victim of retraumatization of their past childhood trauma when those buried emotions erupted. The research proves that the reaction of the suppressed emotions of the past has a negative outburst when it reappears in the living present. The depiction of the trauma in the novel stands as a lead to the idea that how a person's childhood molds them and has a great impact on the personality that they uphold in their adulthood.

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### A PROCEDURE FOR SOLVING MULTI OBJECTIVE HEXAGONAL HESITANT FUZZY FRACTIONAL TRANSPORTATION PROBLEM

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#### ABSTRACT

In this paper, a new algorithm proposed to find the optimal solution for Multi Objective Hexagonal Hesitant Fuzzy Fractional Transportation Problem (MOHFFTP). This

proposed algorithm is very convenient and easy to solve and also this gives the more better solution in both Crisp and Fuzzy environment. The numerical example demonstrated using the proposed algorithm by using the score functions for hexagonal hesitant fuzzy element.

**Keywords:**

Multi objective transportation problem, Hexagonal hesitant set, Score function.

**1. INTRODUCTION**

In the context of globalisation and economic progress, transportation is crucial to our daily lives. The transportation problem is the type of distributive problem, the main goal of the problem is to transfer the goods from various sending location ( i.e) from various sources) to various receiving locations (i.e)to various destinations ) This type of transportation problem is used to minimize the cost and to maximize the profit.

The transportation problem was first studied by Hitchcock in 1941. In the traditional transportation issue, there is only one objective at a time; however, in many real-world scenarios, there are several objectives in addition to overall cost. The idea of the multi-objective transportation problem (MOTP) is thus introduced. In order to address the multi-objective transportation problem, Lee (1973) created the goal programming approach. Zeleny (1974) came up with a non-dominated fundamental viable solution to the multi-objective transportation issue. The algorithm to find every non-dominated solution for MOTP was created by Diaz in 1978. Aneja (1979) and Gupta (1983) are only two of the several authors that have created different methods for solving the MOTP.

Fractional programming problem (FPP) is an extension of linear programming issue where the goals are the ratios of two functions. The optimization of the cost

function ratio is the desired outcome of this. FPP is very useful in a variety of real-world scenarios, including the ratios of profit to time, profit to cost, and inventory to sales, among others. Several methods have been developed by various authors to handle the fractional transportation issue with multiple objectives. These include Charnes and Cooper (1962), Borza (2012), and Chakraborty (2002).

The statistics for a transportation system, such as availabilities, demands, and conveyance capacity, might be hazy, arbitrary, or both due to a lack of knowledge, insufficient data, lack of proof, and other factors. Zadeh (1965) was the first to develop fuzzy sets. In 2009, Torra and Narukawa presented the idea of a Hesitant Fuzzy Set (HFS). Torra (2010) provided an accurate definition of an HFS. Dual- Hesitant Fuzzy Sets (DHFSs), a fuzzy set extension of HFSs, were presented by Zhu (2012).

Zhu et al. and Torra (2010) presented the fundamental characteristics of DHFSs. A goal programming technique for multi-objective fractional transportation issue has been extended by Pop (2007). An approach for handling multi-objective transportation problems in fuzzy environments has been developed by Sheema (2017). Additionally, Gurupada et al.(2019)developed the arithmetic operations on dual hesitant fuzzy numbers, while Amit Kumar et al. (2020) provided a novel ranking approach for dual hesitant fuzzy elements. According to Nayagam & Murugan (2021) , hexagonal fuzzy numbers are superior to trapezoidal in terms of fuzziness and information loss. Merline(2021) reported quantitative results for hesitant fuzzy transportation problems in multi stages. Saranya (2021) demonstrated that the mathematical model of quadratic hesitant fuzzy transportation problem using trapezoidal hesitant fuzzy numbers.

## 2. PRELIMINARIES

### 2.1 HESITANT FUZZY SET

A hesitant fuzzy set (G) on X is defined as a function g(x) that yields a subset of values in the interval [0, 1] when applied to X (i.e.) element of X's power set.

$$g: X \rightarrow \rho([0,1])$$

Mathematically it can be represented as  $G = \{(x_i, g(x_i)) : x_i \in X\}$  Where g(x) is the set of values in [0,1], in addition each component of g(x) is known as hesitant fuzzy element that is denoted as  $x_i$ .

### 2.2 HEXAGONAL HESITANT FUZZY NUMBER

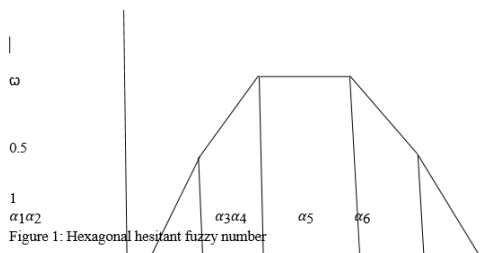
Consider Y to be the firm set, the explanation of a hexagon hesitant fuzzy set on Y is

$$\mathcal{G} = \{ \langle x, F_j g(x) \rangle : x \in Y \}$$

Where  $F_j(x)$  is a function and  $F_j g(x) = (\alpha^1, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6)$ .

$$\left\{ \begin{array}{ll} \frac{\omega(x-\alpha_1)}{2(\alpha_2-\alpha_1)} & , \text{for } \alpha^1 \leq x \leq \alpha_2 \\ \frac{\omega + \frac{\omega(x-\alpha_2)}{2(\alpha_3-\alpha_2)}}{2} & , \text{for } \alpha^2 \leq x \leq \alpha_3 \\ \frac{\omega - \frac{\omega(x-\alpha_4)}{2(\alpha_5-\alpha_4)}}{2} & , \text{for } \alpha^4 \leq x \leq \alpha_5 \\ \frac{\omega(\alpha_6-x)}{2(\alpha_6-\alpha_5)} & , \text{for } \alpha^5 \leq x \leq \alpha_6 \end{array} \right.$$

0 ,Otherwise



### 2.3 ARITHMETIC OPERATIONS ON HEXAGONAL HESITANT FUZZY SET

Let  $HH_1 = \{ \langle x_i, F_j g_1(x_i) \rangle : x_i \in Y \}$  and  $HH_2 = \{ \langle x_i, F_j g_2(x_i) \rangle : x_i \in Y \}$  be the two hexagonal hesitant fuzzy elements. Then it satisfies some arithmetic operations like Addition, Subtraction, Scalar multiplication and Division,

**ADDITION :**

$$HH_1 + HH_2 = \cup \gamma g_1 \in g_1, \gamma g_2 \in g_2, \delta g_1 \in g_1, \delta g_2 \in g_2$$

$$\{ \{ \gamma g_1 + \gamma g_2 - \gamma g_1 \cdot \gamma g_2 \}, \{ \delta g_1 \cdot \delta g_2 \} \}$$

**SUBTRACTION :**

$$HH_1 - HH_2 = \cup \gamma g_1 \in g_1, \gamma g_2 \in g_2, \delta g_1 \in g_1, \delta g_2 \in g_2$$

$$\{ \{ \delta g_1 - \delta g_2 \}, \{ \delta g_1 + \delta g_2 - \delta g_1 \cdot \delta g_2 \} \}$$

**MULTIPLICATION :**

$$HH_1 \times HH_2 = \cup \gamma g_1 \in g_1, \gamma g_2 \in g_2, \delta g_1 \in g_1, \delta g_2 \in g_2$$

$$\{ \{ \gamma g_1 \cdot \gamma g_2 \}, \{ \delta g_1 \cdot \delta g_2 \} \}$$

### 2.4 SCORE FUNCTION OF HEXAGONAL HESITANT FUZZY SET

Let  $\mathcal{H} = \{ \langle x_i, F_j g(x) \rangle : x_i \in Y \}$  be a hexagonal hesitant fuzzy set where  $(x_1, x_2, x_3, \dots, x_n)$  and  $S = (g_1s, g_2s)$  be a hexagonal hesitant fuzzy elements. Then the score functions will be

$$g_s(x_i) = \gamma_i + \frac{1}{6} \sum (\alpha_i)$$

### 3. MATHEMATICAL FORMULATION OF MULTI OBJECTIVE FUZZY RANSPORTATION PROBLEM

#### NOTATIONS

- $m$  : Total number of Sources
- $n$  : Total number of destinations
- $l_{ikj}$  : Hexagonal hesitant fuzzy numerator cost coefficients
- $m_{ikj}$  : Hexagonal hesitant fuzzy denominator cost coefficient
- $d_{ik}$  : Total number of units transported
- $\alpha_n$  : Hexagonal hesitant fuzzy availability
- $\beta_m$  : Hexagonal hesitant fuzzy demand

Min or Max

$$\text{Min or (max) } z^*_1 = \frac{\sum_{i=1}^m \sum_{k=1}^n l_{ik1} d_{ik}}{\sum_{i=1}^m \sum_{k=1}^n m_{ik1} d_{ik}}$$

$$\frac{\sum_{i=1}^m \sum_{k=1}^n m_{ik1} d_{ik}}$$

$$\text{Min or (max) } z^*_2 = \frac{\sum_{i=1}^m \sum_{k=1}^n l_{ik2} d_{ik}}{\sum_{i=1}^m \sum_{k=1}^n m_{ik2} d_{ik}}$$

$$\frac{\sum_{i=1}^m \sum_{k=1}^n m_{ik2} d_{ik}}$$

$$\text{Min or (max) } z^*_n = \frac{\sum_{i=1}^m \sum_{k=1}^n l_{ikn} d_{ik}}{\sum_{i=1}^m \sum_{k=1}^n m_{ikn} d_{ik}}$$

$$\frac{\sum_{i=1}^m \sum_{k=1}^n m_{ikn} d_{ik}}$$

Subject to constraints

$$\sum_{i=1}^m d_{ik} = \alpha_i$$

$$\sum_{k=1}^n d_{ik} = \beta_i$$

for every  $i = 1$  to  $m$ ,  $k = 1$  to  $n$ ,  $l = 1$  to  $n$ .

Q.

### 4. PROCEDURE FOR SOLVING MULTI OBJECTIVE HEXAGONAL HESITANT FRACTIONAL FUZZY TRANSPORTATIONAL PROBLEM

Step 1: Find the score value for every hexagonal hesitant fuzzy number by using the ranking function or score functions defined in preliminaries.

Step 2: Find the Fractional values of the each rows and columns.

Step 3: If all the objectives are seem to be maximum then it can be converted into type of minimization by subtracting the greatest element from all the fractional values.

Step 4: Find the greatest ratio of every row  $\lambda_{ij}$  and every column  $\eta_{kl}$  and fix

Step 5: Select  $\Omega = \{\lambda_{ij}, \eta_{kl}\}$  for each  $i=1$  to  $m, k=1$  to  $n, l=1$  to  $p$ .

Step 6: Choose the cell containing  $\Omega$  as one of its ratio. in case, there exists more than one cells select the maximum ratio for other fractional objectives.

Step 7: Now select the cell having the  $\min \left\{ \frac{\sum_{i=1}^m \sum_{k=1}^n l_{ikn}}{d_{ik}} \right\}$  for fixed  $i, l$  at any two  $\frac{\sum_{i=1}^m \sum_{k=1}^n m_{ikn} d_{ik}}$

### 5. NUMERICAL EXAPMLE

A numerical example employing the suggested approach is solved to demonstrate its efficacy. Think about the multi-objective fuzzy fractional transportation issue that follows. Three goals are taken into consideration here. The first goal is transportation cost, which is calculated as the difference between actual and preferred costs. The second goal relates to travel time, which is calculated as the difference between the desired and actual travel times. The third goal has to do with damage cost, which is calculated as the difference between actual and preferred costs. Every problem parameter is represented as a hexagonal hesitant fuzzy number.

**Table 1. Transportation Cost**

	M	N	O	SUPPLY
<b>P</b>	<u>12:0.5,0.4,0.3,</u> <u>0.7,0.6,0.2</u> 8:0.3,0.2,0.1,0. 9,0.7,0.3	<u>(11:0.6,0.3,0.2,0</u> <u>4,0.1,0.5)</u> (9:0.9,0.7,0.5,0. 3,0.2,0.4)	<u>(28:0.2,0.4,</u> <u>0.6,0.1,0.5,0</u> <u>.7)</u> (24:0.7,0.4, 0.9,0.2,0.5,0. .3)	(10:0.6,0.2,0. 4,0.3,0.1,0.2)
<b>Q</b>	<u>(13:0.6,0.4,0.5,</u> <u>0.3,0.9,0.2)</u> (18 :0.1,0.2,0.3,0.4 ,0.5,0.6)	<u>(9:0.8,0.7,0.6,0.</u> <u>5,0.4,0.3)</u> (4:0.7, 0.6,0.9,0.5,0.2,0. 4)	<u>(18:0.25,0.3</u> <u>.0.5,0.8,0.4,</u> <u>0.2)</u> (6:0.75, 0.6,0.3,0.5,0. .8,0.2)	(12:0.6,0.7,0. 9,0.85,0.8,0.4 )
<b>R</b>	<u>(24:0.2,0.4,0.3</u> <u>5,0.6,0.9,0.3)</u> (16:0.8,0.9,0.7 5,0.5,0.4,0.3)	<u>(29:0.4,0.2,0.1,0</u> <u>.6,0.5,0.3)</u> (15:0.65,0.2,0.3, 0.6,0.9,0.7)	<u>(16:0.6,0.3,</u> <u>0.25,0.9,0.7,</u> <u>0.2)</u> (20:0.5,0.25 .0.7,0.6,0.5, 0.3)	(16:0.3,0.5,0. 7,0.4,0.6,0.1)
<b>DEMAND</b>	(8:0.2,0.4,0.6,0 .8,0.1,0.3)	(10:0.1,0.6,0.5,0 .7,0.9,0.2)	(20:0.75,0.4 .0.7,0.65,0.9 .0.35)	

**Table 2: Transportation Time**

	M	N	O	SUPPLY
<b>P</b>	<u>(15:0.7,0.25,0.4,0.5,0</u> <u>.6,0.9)</u> (20:0.75,0.3,0.6,0.9,0 .8,0.2)	<u>(30:0.5,0.4,0.3,0.9,0.8,</u> <u>0.2)</u> (28:0.2,0.6,0.8,0.9,0.7, 0.5) -	<u>(22:0.75,0.3,0.25</u> <u>.0,4,0.8,0.5)</u> (34:0.6,0.5,0.35, 0.4,0.2,0.1)	(10:0.6, 0.2,0.4, 0.3,0.1, 0.2)
<b>Q</b>	<u>(3:0.7,0.9,0.5,0.35,0.</u> <u>25,0.1)</u> (6:0.25,0.4,0.35,0.8,0 .45,0.6)	<u>(20:0.1,0.7,0.25,0.9,0.3</u> <u>.0,4)</u> (8:0.9,0.35,0.4,0.8,0.15 .0.2)	<u>(12:0.6,0.5,0.3,0.</u> <u>25,0.15,0.9)</u> (10:0.1,0.2,0.3,0. 4,0.6,0.8)	(12:0.6, 0.7,0.9,0 .85,0.8, 0.4)
<b>R</b>	<u>(30:0.65,0.2,0.35,0.4,</u> <u>0.6,0.9)</u> (25:0.1,0.35,0.5,0.75, 0.8,0.3)	<u>(19:0.3,0.6,0.9,0.8,0.68</u> <u>.0.7)</u> (13:0.1,0.25,0.4,0.65,0. 7,0.8)	<u>(35:0.7,0.5,0.4,0.</u> <u>2,0.1,0.3)</u> (14:0.9,0.7,0.8,0. 6,0.2,0.1)	(16:0.3, 0.5,0.7, 0.4,0.6, 0.1)
<b>DEMAND</b>	(8:0.2,0.4,0.6,0.8,0.1, 0.3)	(10:0.1,0.6,0.5,0.7,0.9, 0.2)	(20:0.75,0.4,0.7, 0.65,0.9, 0.35)	

**Table 3. Damage Cost**

Using the proposed algorithm, the optimum solution of given multi objective, hesitant fractional transportation problem are

$$z_1 = \frac{(830:0.6,0.2,0.4,0.3,0.1,0.2)}{(490:0.5,0.4,0.2,0.9,0.7,0.15)}; z_2 = \frac{(832:0.55,0.4,0.8,0.2,0.1,0.25)}{(616:0.05,0.3,0.1,0.65,0.7,0.15)}; z_3 = \frac{(1228:0.15,0.2,0.3,0.35,0.8,0.15)}{(978:0.15,0.3,0.4,0.05,0.2,0.0,0.05)}$$

**6. CONCLUSION**

This study proposes a novel approach for optimally solving the multi-objective Hexagonal hesitant fuzzy transportation problem. The suggested approach is straightforward and easy to understand. This method provides the best answer in a hexagonal hesitant fuzzy environment. A numerical example is solved to demonstrate the algorithm. The suggested method's key benefit is its use of hesitant fuzzy elements to solve the problem. The suggested algorithm outperforms the existing solution. Thus the solution also in the form of hexagonal hesitant fuzzy element which is main advantage of this proposed algorithm.

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